

Math Analysis Formula Sheet

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \quad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \quad \sec\left(\frac{\pi}{2} - u\right) = \csc u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \quad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

Even/Odd Identities

$$\sin(-u) = -\sin u \quad \csc(-u) = -\csc u$$

$$\cos(-u) = \cos u \quad \sec(-u) = \sec u$$

$$\tan(-u) = -\tan u \quad \cot(-u) = -\cot u$$

Sum & Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Half-Angle Formulas

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2} [\cos(u-v) - \cos(u+v)]$$

$$\cos u \cos v = \frac{1}{2} [\cos(u-v) + \cos(u+v)]$$

$$\sin u \cos v = \frac{1}{2} [\sin(u+v) + \sin(u-v)]$$

$$\cos u \sin v = \frac{1}{2} [\sin(u+v) - \sin(u-v)]$$

Second Semester Formulas

DeMoivre's Theorem

(Finding Powers of Complex Numbers)

If $z = r(\cos \theta + i \sin \theta)$ is a complex number and n is a positive integer, then

$$z^n = r^n (\cos n\theta + i \sin n\theta)$$

n^{th} Roots of a Complex Number

For a positive integer n , the complex number $z = r(\cos \theta + i \sin \theta)$ has EXACTLY n distinct n^{th} roots given by:

$$\sqrt[n]{r} \left(\cos \frac{\theta + 2\pi k}{n} + i \sin \frac{\theta + 2\pi k}{n} \right)$$

Angle between two vectors

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$$

Projection of \mathbf{u} onto \mathbf{v}

$$\text{proj}_{\mathbf{v}} \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v}$$

Standard Equation of an Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$