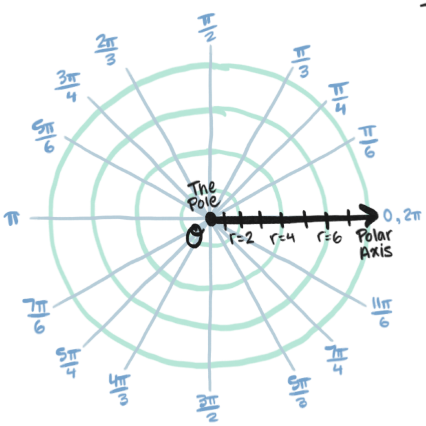
9.5 Polar Coordinates

Objective: Today we will plot POLAR COORDINATES & convert points & equations from polar-to-rectangular and vice versa.

Warm-up: What is the length of the Radius in a Unit Circle? What do the x- & y-coordinates represent?

9.5 POLAR COORDINATES



To form the POLAR COORDINATE SYSTEM in the plane,

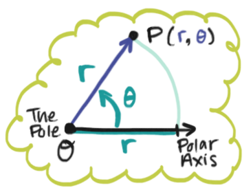
① Fix a point O, called the $\boxed{\text{POLE}}$ ← (the former Origin), &

② CONSTRUCT a RAY from O called the $\boxed{\text{POLAR AXIS}}$. ← (old $+x$-axis)

Then ③ each Point P can be assigned the polar coordinates $\boxed{(r, \theta)}$ where:

① r = DIRECTED distance from O to P.

② θ = DIRECTED Angle, CCW from the Polar Axis to $\overline{OP}$.

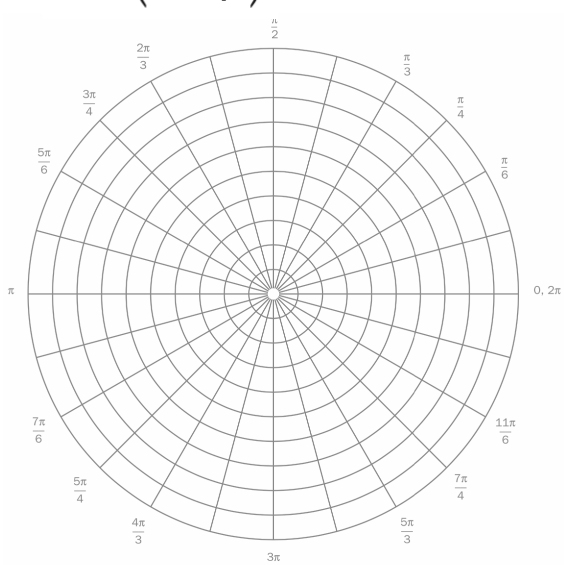


COORDINATE CONVERSIONS

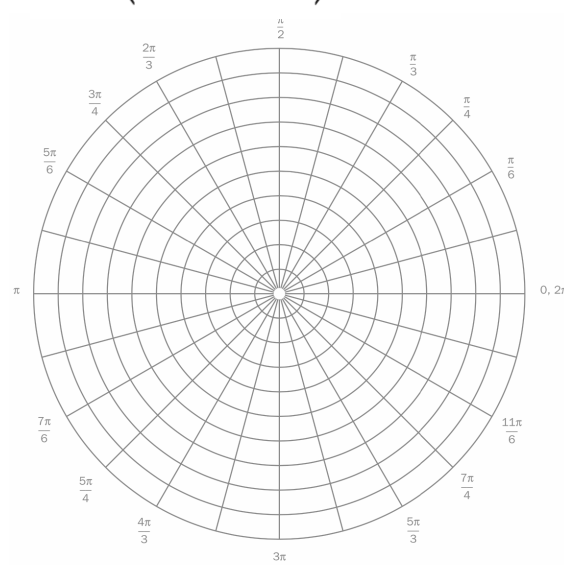
POLAR → RECTANGULAR	RECTANGULAR → POLAR
$(r,\theta) \Longrightarrow (x,y)$	$(x,y) \Longrightarrow (r,\theta)$
$\boxed{x = r\cos\theta}$ &	$\tan\theta = \frac{y}{x}$ & $r^2 = x^2 + y^2$
$\boxed{y = r\sin\theta}$	$\boxed{\theta = \tan^{-1}\left(\frac{y}{x}\right)}$ $\boxed{r = \sqrt{x^2+y^2}}$

Plotting Points in the Polar Coordinate System In Exercises 9–16, plot the point given in polar coordinates and find three additional polar representations of the point, using $-2\pi < \theta < 2\pi$.

10. $\left(2, \frac{3\pi}{4}\right)$

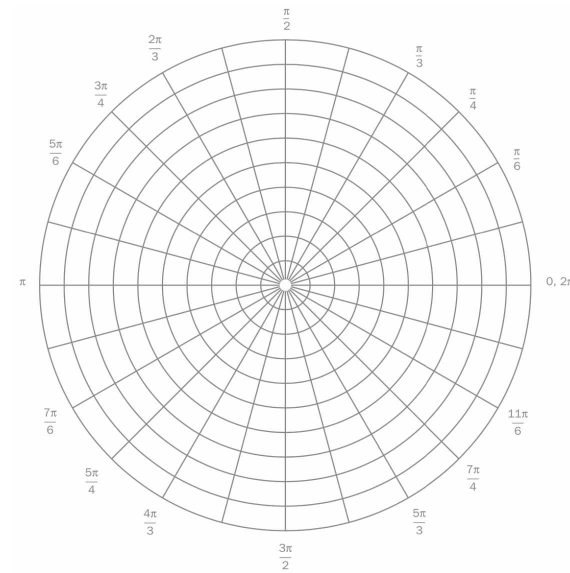


12. $\left(-3, -\frac{7\pi}{6}\right)$



Polar-to-Rectangular Conversion In Exercises 17–26, plot the point given in polar coordinates and find the corresponding rectangular coordinates for the point.

20. $\left(16, \dfrac{5\pi}{2}\right)$



Checkpoint: Plot the polar coordinate & find its rectangular representation.

$$(-4, 210°)$$

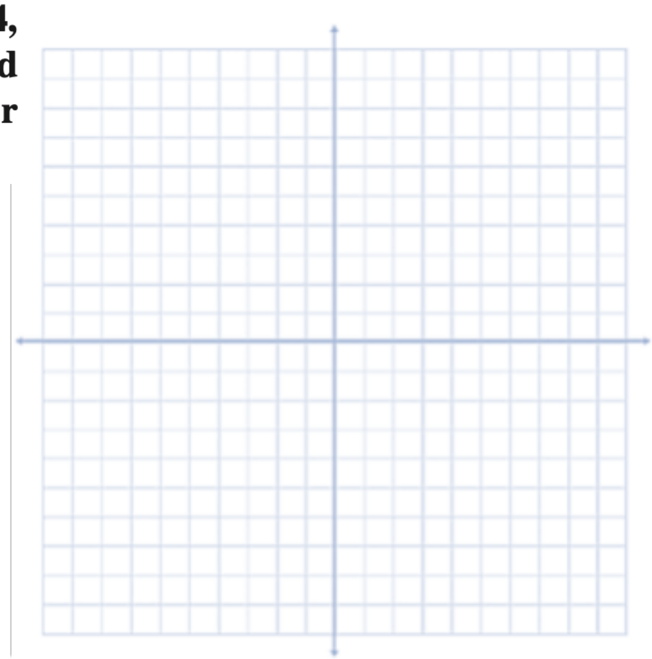
Using a Graphing Utility to Find Rectangular Coordinates

In Exercises 27–34, use a graphing utility to find the rectangular coordinates of the point given in polar coordinates. Round your results to two decimal places.

30. $(8.25, 3.5)$

Rectangular-to-Polar Conversion In Exercises 35–44, plot the point given in rectangular coordinates and find two sets of polar coordinates for the point for $0 \le \theta < 2\pi$.

44. $(5, 12)$



Converting a Rectangular Equation to Polar Form In Exercises 51–68, convert the rectangular equation to polar form. Assume $a > 0$.

52. $x^2 + y^2 = 16$

64. $x^2 + y^2 - 8y = 0$

Converting a Polar Equation to Rectangular Form In Exercises 69–88, convert the polar equation to rectangular form.

70. $r = 2\cos\theta$

76. $\theta = \pi$

84. $r = 3\cos 2\theta$

86. $r = \dfrac{2}{1 + \sin\theta}$

HINT for #83

83 $r = 2\sin(3\theta)$

use the Sum & Difference Formula

$r = 2\sin[2\theta + \theta]$

$r = 2[\sin(2\theta)\cos\theta + \cos(2\theta)\sin\theta]$

Use this one

$\boxed{\sin(2\theta) = 2\sin\theta\cos\theta}$ $\boxed{\cos(2\theta) = 2\cos^2\theta - 1}$

$= \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

Double Angle Formulas (From Section 5.5)

Make these substitutions & then keep going until no r, θ, or $\sqrt{\ }$.