7.3 Multivariable Linear Systems

Objective: Today we will solve systems of multivariable equations.

Warm-up: Which method for solving the system is best? Is the system CONSISTENT or INCONSISTENT?

1. $\begin{cases} x + 7y = 12 \\ 3x - 5y = 10 \end{cases}$

2. $\begin{cases} 1.8x + 1.2y = 4 \\ 9x + 6y = 3 \end{cases}$

3. $\begin{cases} x + 5y = 10 \\ 3x - 10y = -5 \end{cases}$

7.3 Multivariable Linear Systems

NOTE:

① CONSISTENT = AT LEAST ONE Solution
- INDEPENDENT = EXACTLY ONE Sol.
- DEPENDENT = INFINITELY Many Sol.s

② INCONSISTENT = NO SOLUTION

ROW-ECHELON FORM & BACK-SUBSTITUTION

- The Elimination Method can be used to SOLVE Systems of LINEAR Equations & Back-Substitution is applied to Solve for remaing variables.
- ROW-ECHELON FORM means that the system has a "Stair-step" pattern with Leading Coefficients of 1.

ex. System in 3 Variables

$$\begin{cases} x - 2y + 3z = 9 \\ -x + 3y + z = -2 \\ 2x - 5y + 5z = 17 \end{cases}$$

EQUIVALENT SYSTEM in ROW-ECHELON FORM

$$\begin{cases} x - 2y + 3z = 9 \\ y + 4z = 7 \\ z = 2 \end{cases}$$

GAUSSIAN ELIMINATION

- The Process of Converting a system into ROW-ECHELON FORM using Elementary Row operations is called GAUSSIAN Elimination.

ELEMENTARY ROW OPERATIONS

① INTERCHANGE Two Equations ← Switch two Rows

② MULTIPLY one Equation by a NON zero Constant. ← Scale a Row

③ ADD a multiple of one Equation to another. ← Combine two Rows

Checking Solutions In Exercises 9–12, determine whether each ordered triple is a solution of the system of equations.

10. $\begin{cases} 3x + 4y - z = 17 \\ 5x - y + 2z = -2 \\ 2x - 3y + 7z = -21 \end{cases}$

(a) $(1, 5, 6)$ (b) $(-2, -4, 2)$

(c) $(1, 3, -2)$ (d) $(0, 7, 0)$

Using Back-Substitution **In Exercises 13–18, use back-substitution to solve the system of linear equations.**

16. $$\begin{cases} x - y + 2z = 22 \\ \quad 3y - 8z = -9 \\ \qquad\quad z = -3 \end{cases}$$

Performing Row Operations **In Exercises 19 and 20, perform the row operation and write the equivalent system. What did the operation accomplish?**

20. Add -2 times Equation 1 to Equation 3.

$$\begin{cases} x - 2y + 3z = 5 & \text{Equation 1} \\ -x + 3y - 5z = 4 & \text{Equation 2} \\ 2x \qquad - 3z = 0 & \text{Equation 3} \end{cases}$$

Solving a System of Linear Equations **In Exercises 21–42, solve the system of linear equations and check any solution algebraically.**

22. $$\begin{cases} x + y + z = 3 \\ x - 2y + 4z = 5 \\ \quad 3y + 4z = 5 \end{cases}$$

36. $$\begin{cases} 3x - 2y - 6z = -4 \\ -3x + 2y + 6z = 1 \\ x - y - 5z = -3 \end{cases}$$

Checkpoint: Solve the system of linear equations.

$$\begin{cases} 5x \qquad - 8z = 22 \\ \quad 3y - 5z = 10 \\ \qquad\quad z = -4 \end{cases}$$

DECOMPOSITION OF RATIONAL FUNCTIONS INTO PARTIAL FRACTIONS

① **DIVIDE** if improper:

$\frac{N(x)}{D(x)}$ is IMPROPER if the (deg. of $N(x)$) $\geq$ (deg. of $D(x)$)

$$\frac{N(x)}{D(x)} = (\text{polynomial}) + \frac{N_1(x)}{D_1(x)}$$

- Then APPLY STEPS 2, 3, & 4 to the PROPER rational expression $\left(\frac{N_1(x)}{D_1(x)}\right)$.

② **FACTOR** Denominator:

- Completely factor the denominator into factors of the form

$\boxed{(px+q)^m}$ and $\boxed{(ax^2+bx+c)^n}$.

Linear Factors — Irreducible quadratic factors

③ FOR EACH **LINEAR** FACTOR: $(px+q)^m$

The PARTIAL FRACTION DECOMPOSITION is the sum of m fractions:

$$\frac{A_1}{px+q} + \frac{A_2}{(px+q)^2} + \cdots + \frac{A_m}{(px+q)^m}$$

④ FOR EACH **QUADRATIC** Factor: $(ax^2+bx+c)^n$

The PARTIAL FRACTION DECOMPOSITION is the sum of n fractions:

$$\frac{B_1x+C_1}{ax^2+bx+c} + \frac{B_2x+C_2}{(ax^2+bx+c)^2} + \cdots + \frac{B_nx+C_n}{(ax^2+bx+c)^n}$$

Writing the Partial Fraction Decomposition In Exercises 51–56, write the form of the partial fraction decomposition of the rational expression. Do not solve for the constants.

52. $\dfrac{x-2}{x^2+4x+3}$

Partial Fraction Decomposition In Exercises 57–70, write the partial fraction decomposition for the rational expression. Check your result algebraically by combining fractions.

62. $\dfrac{x-2}{x^2+4x+3}$

Partial Fraction Decomposition In Exercises 57–70, write the partial fraction decomposition for the rational expression. Check your result algebraically by combining fractions.

64. $\dfrac{x^2 + 12x - 9}{x^3 - 9x}$

Finding the Equation of a Circle In Exercises 81–84, find the equation of the circle

$x^2 + y^2 + Dx + Ey + F = 0$

82. $(0, 0), (0, 6), (3, 3)$

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86. Finance A small corporation borrowed \$800,000 to expand its line of toys. Some of the money was borrowed at 8%, some at 9%, and some at 10%. How much was borrowed at each rate given that the annual interest was \$67,000 and the amount borrowed at 8% was five times the amount borrowed at 10%?