

# 6.3 Vectors in the Plane

**Objective:** Today we will find the magnitude of vectors, write vectors in their component form, and perform operations on vectors.

**Warm-up:** Solve the triangle for its missing parts.

$$A = 60^\circ \quad b = 3 \quad c = 6$$

## 6.3 VECTORS IN THE PLANE

The directed line segment  $\vec{PQ}$  is called a **VECTOR** with the initial point P & terminal point Q.

The **MAGNITUDE**, or length, of  $\vec{PQ}$  is denoted by  $\|\vec{PQ}\|$  & can be found using the Distance Formula.

$\vec{PQ}$   $Q(q_1, q_2)$   
P  $(p_1, p_2)$

Two Vectors that have  
① the SAME Magnitude &  
② the SAME Direction are  
**EQUIVALENT**

unsimplified slope

The **COMPONENT FORM** of a vector  $\vec{PQ}$  is given by

$$\vec{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle = \vec{v} = v\hat{i}$$

Sometimes a vector is denoted by a boldfaced letter.

$$\|\vec{v}\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = \sqrt{v_1^2 + v_2^2}$$

Let  $\vec{u} = \langle u_1, u_2 \rangle$  &  $\vec{v} = \langle v_1, v_2 \rangle$  be vectors & let k be a scalar.

① **SUM:**  
 $\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$

② **Scalar Multiple:**  
 $k\vec{u} = \langle ku_1, ku_2 \rangle$

**PROPERTIES OF VECTOR ADDITION & SCALAR MULTIPLICATION**

①  $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

②  $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$

③  $\vec{u} + \vec{0} = \vec{u}$     ④  $\vec{u} + (-\vec{u}) = \vec{0}$

⑤  $c(d\vec{u}) = (cd)\vec{u}$     Note:  $\vec{0} = \langle 0, 0 \rangle$  (the zero vector)

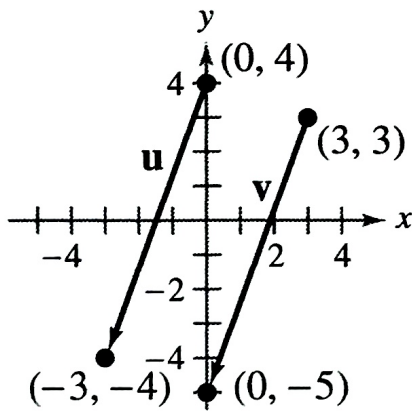
⑥  $(c+d)\vec{u} = c\vec{u} + d\vec{u}$

⑦  $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$

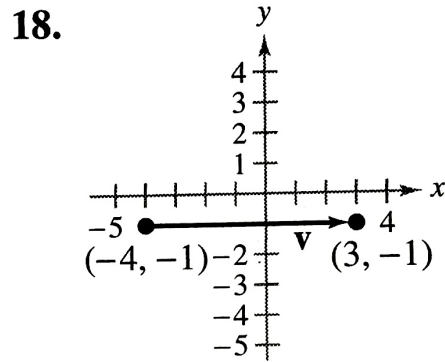
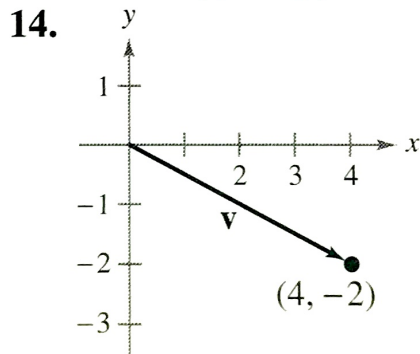
⑧  $k\vec{u} = \vec{u}, 0\vec{u} = \vec{0}$     ⑨  $\|c\vec{v}\| = |c|\|\vec{v}\|$

**Equivalent Directed Line Segments** In Exercises 11 and 12, show that  $\vec{u} = \vec{v}$ .

12.



**Finding the Component Form of a Vector** In Exercises 13–24, find the component form and the magnitude of the vector  $\mathbf{v}$ .




---

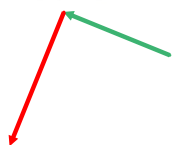
**Finding the Component Form of a Vector** In Exercises 13–24, find the component form and the magnitude of the vector  $\mathbf{v}$ .

20. *Initial Point*  
 $(-3, 11)$

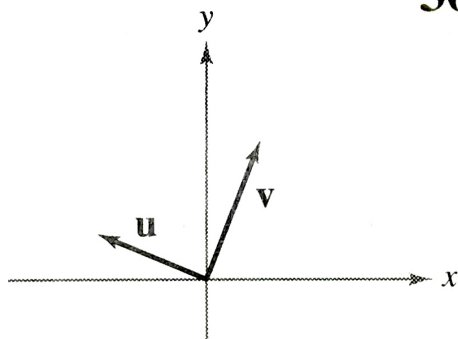
*Terminal Point*  
 $(9, 40)$

**Sketching the Graph of a Vector** In Exercises 25–30, use the figure to sketch a graph of the specified vector. To print an enlarged copy of the graph, go to the website

28.  $\mathbf{u} - \mathbf{v}$

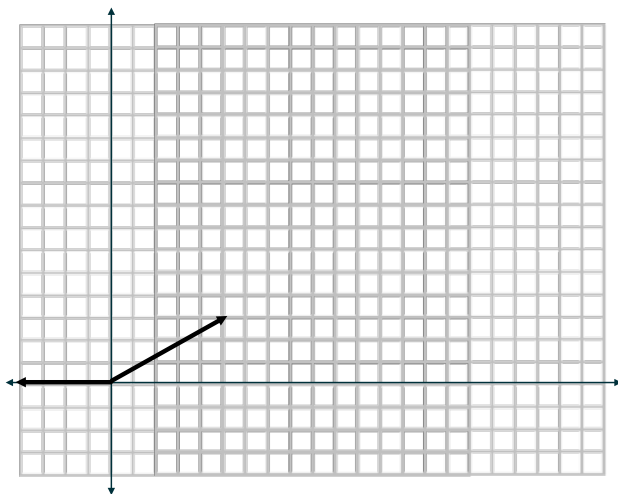


30.  $\mathbf{v} - \frac{1}{2}\mathbf{u}$



**Vector Operations** In Exercises 37–42, find (a)  $\mathbf{u} + \mathbf{v}$ , (b)  $\mathbf{u} - \mathbf{v}$ , (c)  $2\mathbf{u} - 3\mathbf{v}$ , and (d)  $\mathbf{v} + 4\mathbf{u}$ . Then sketch each resultant vector.

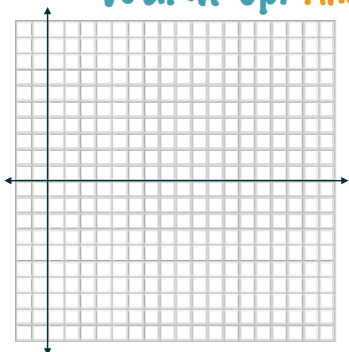
38.  $\mathbf{u} = \langle 5, 3 \rangle$ ,  $\mathbf{v} = \langle -4, 0 \rangle$



# 6.3 Vectors in the Plane

**Objective:** Today we will find the magnitude of vectors, write vectors in their component form, and perform operations on vectors.

**Warm-up:** Find and graph the resultant vectors.



$$\mathbf{u} = \langle 6, 0 \rangle \quad \mathbf{v} = \langle 8, -2 \rangle$$

1.  $3\mathbf{u} - 2\mathbf{v}$

2.  $-\mathbf{u} + 3\mathbf{v}$

**UNIT VECTORS**

- A **UNIT VECTOR**,  $\hat{u}$ , is a vector that has a magnitude of 1. ( $\|\hat{u}\| = 1$ ).
- UNIT VECTOR** in the direction of  $\vec{v}$ :  

$$\hat{u} = \frac{\vec{v}}{\|\vec{v}\|} = \left(\frac{1}{\|\vec{v}\|}\right)\vec{v}$$

*To FIND ANY vector  $\vec{v}$  in the SAME direction as  $\vec{w}$ :*  

$$\vec{v} = \frac{\|\vec{v}\|}{\|\vec{w}\|} \vec{w}$$

**NOTE:**

$\vec{v} = \langle v_1, v_2 \rangle$   
Component Form

$\vec{v} = v_1 \langle 1, 0 \rangle + v_2 \langle 0, 1 \rangle$   
Linear Combination Form

$\vec{v} = v_1 \hat{i} + v_2 \hat{j}$

**DIRECTION ANGLES**

- Let  $\theta$  be the direction angle of  $\vec{u}$ .  

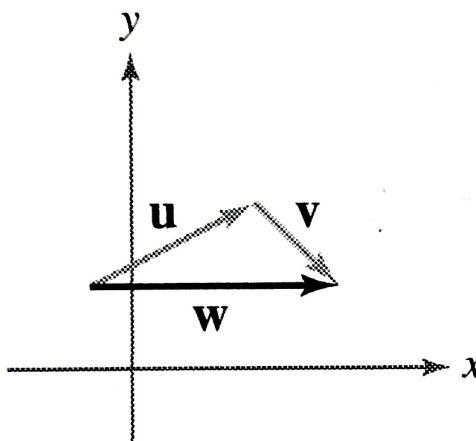
$$\vec{u} = \langle x, y \rangle = \langle \cos \theta, \sin \theta \rangle = \cos \theta \hat{i} + \sin \theta \hat{j}$$
- If  $\vec{v} = a\hat{i} + b\hat{j}$  is ANY vector in the SAME direction as  $\vec{u}$ , then  

$$\vec{v} = \|\vec{v}\| \langle \cos \theta, \sin \theta \rangle = \|\vec{v}\| \cos \theta \hat{i} + \|\vec{v}\| \sin \theta \hat{j}$$

**Writing a Vector** In Exercises 43–46, use the figure and write the vector in terms of the other two vectors.

44.  $\mathbf{v}$

46.  $2\mathbf{v}$



**Finding a Unit Vector** In Exercises 47–56, find a unit vector in the direction of the given vector. Verify that the result has a magnitude of 1.

50.  $\mathbf{v} = \langle 3, -4 \rangle$

52.  $\mathbf{v} = \langle 8, -20 \rangle$

---

**Finding a Vector** In Exercises 57–62, find the vector with the given magnitude and the same direction as  $\mathbf{u}$ .

	<i>Magnitude</i>	<i>Direction</i>
58.	$\ \mathbf{v}\  = 3$	$\mathbf{u} = \langle 4, -4 \rangle$

	<i>Magnitude</i>	<i>Direction</i>
60.	$\ \mathbf{v}\  = 10$	$\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}$

**Writing a Linear Combination of Unit Vectors** In Exercises 63–66, the initial and terminal points of a vector are given. Write the vector as a linear combination of the standard unit vectors  $\mathbf{i}$  and  $\mathbf{j}$ .

<i>Initial Point</i>	<i>Terminal Point</i>
65. $(-1, -5)$	$(2, 3)$

---

**Finding Direction Angles of Vectors** In Exercises 73–78, find the magnitude and direction angle of the vector  $\mathbf{v}$ .

74.  $\mathbf{v} = 8(\cos 135^\circ \mathbf{i} + \sin 135^\circ \mathbf{j})$

78.  $\mathbf{v} = 12\mathbf{i} + 15\mathbf{j}$