

3.5 Exponential & Logarithmic Models

Objective: Today we will use nonlinear functions to model data.

Warm-up: Solve for x , and check for extraneous solutions.

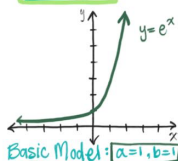
$$\log_5 x + \log_5 (x - 8) = \log_{125} 729$$

3.5 EXPONENTIAL & LOGARITHMIC MODELS

MOST COMMON MODELS
using Exponential & Logarithmic Functions:

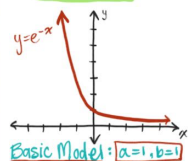
① **EXPONENTIAL GROWTH**

$$y = ae^{bx}, b > 0$$



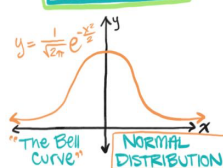
② **EXPONENTIAL DECAY**

$$y = ae^{-bx}, b > 0$$



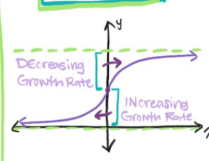
③ **GAUSSIAN**

$$y = ae^{-(x-b)^2/c}$$



④ **LOGISTIC GROWTH**

$$y = \frac{a}{1 + be^{-rx}}$$

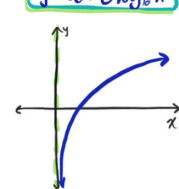


⑤ **LOGARITHMIC**

$$y = a + b \ln x$$

OR

$$y = a + b \log_b x$$



Using a Compound Interest Formula In Exercises 13–20, complete the table for a savings account in which interest is compounded continuously.

Initial Investment	Annual % Rate	Time to Double	Amount After 10 Years
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14. \$2000

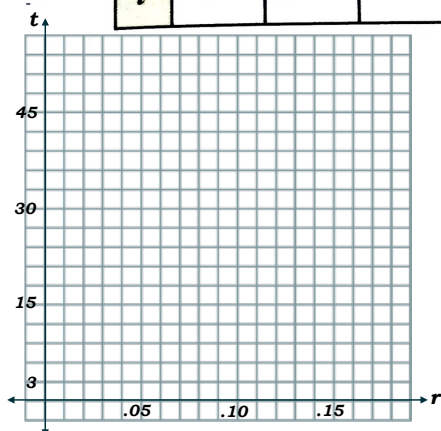
1.5%

16. \$1000

12 years

22. **Tripling an Investment** Complete the table for the time t (in years) necessary for P dollars to triple when interest is compounded annually at rate r . Create a scatter plot of the data.

r	2%	4%	6%	8%	10%	12%
t						



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36. **Demography** The populations P (in thousands) of Raleigh, North Carolina from 2000 through 2008 can be modeled by $P = 289.81e^{kt}$, where t is the year, with $t = 0$ corresponding to 2000. In 2006, the population was 363,000. (Source: U.S. Census Bureau)
- (a) Find the value of k for the model. Round your result to four decimal places.
- (b) Use your model to predict the population in 2015.

44. **Biology** The number Y of yeast organisms in a culture is given by the model

$$Y = \frac{663}{1 + 72e^{-0.547t}}, \quad 0 \leq t \leq 18$$

where t represents the time (in hours). See margin.

- Use a graphing utility to graph the model.
- Use the model to predict the populations for the 19th hour and the 30th hour.
- According to this model, what is the limiting value of the population?
- Why do you think this population of yeast follows a logistic growth model instead of an exponential growth model?

3.6 Nonlinear Models

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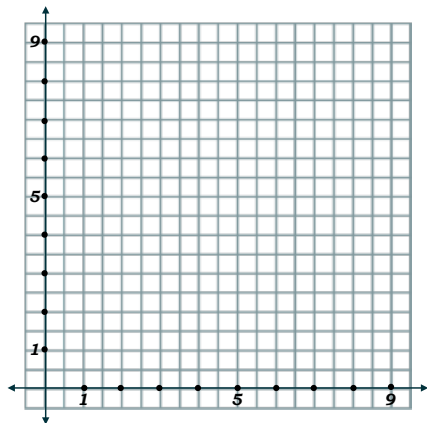
Checkpoint: If you invest P dollars in an account at an annual rate of 5.17%. How long would it take for your money to QUADruple if it compounds annually?

3.6 NONLINEAR MODELS



Classifying Scatter Plots In Exercises 13–18, use a graphing utility to create a scatter plot of the data. Decide whether the data could best be modeled by a linear model, an exponential model, or a logarithmic model.

18. $(1, 5.0)$, $(1.5, 6.0)$, $(2, 6.4)$, $(4, 7.8)$, $(6, 8.6)$, $(8, 9.0)$



32. MODELING DATA

The table shows the annual amounts A (in billions of dollars) spent in the U.S. by the cruise lines and passengers of the North American cruise industry from 2003 through 2008. (Source: Cruise Lines International Association) See margin.



Year	Amount, A
2003	12.92
2004	14.70
2005	16.18
2006	17.64
2007	18.70
2008	19.07

- Use the *regression* feature of a graphing utility to find a linear model, an exponential model, and a logarithmic model for the data. Let t represent the year, with $t = 3$ corresponding to 2003.
- Use the graphing utility to graph each model with the data. Use the graphs to determine which model best fits the data.
- Use the model you chose in part (b) to predict the amount spent in 2009. Is the amount reasonable?