

2.2 Polynomials of a Higher Degree

Objective: Today we will compare polynomial functions and use the **Leading Coefficient Test** to determine a functions end behavior.

Warm-up: Write the equation of the parabola whose vertex is given and goes through the given point.

Vertex: $(-3, 5)$ **Point:** $(5, 17)$

2.2 POLYNOMIALS OF A HIGHER DEGREE

The graph of a Polynomial function is **CONTINUOUS**

IF it has:

- NO BREAKS
- NO HOLES
- NO GAPS

AND

the graph has **SMOOTH/ROUNDED Turns**.

POWER FUNCTIONS $f(x) = x^n$

FOR $f(x) = x^n$

- the GREATER the value of n is, the FLATTER it is by the origin.
- $n = \text{EVEN}$, the graph is SIMILAR to the graph of $f(x) = x^2$.
- $n = \text{ODD}$, the graph is SIMILAR to the graph of $f(x) = x^3$.

NOTE: the LEADING COEFFICIENT TEST determines the END BEHAVIOR of the graph.

LEADING COEFFICIENT TEST

For $f(x) = a_n x^n + \dots + a_1 x + a_0$,

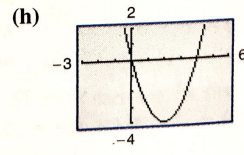
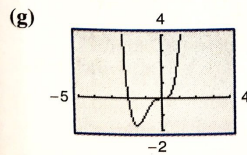
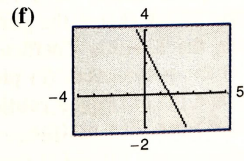
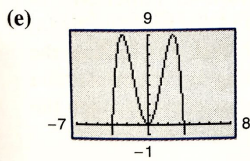
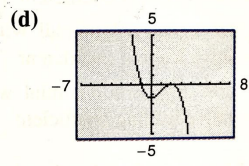
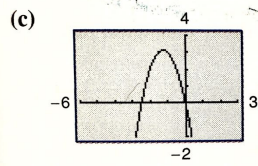
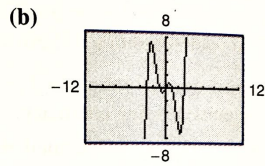
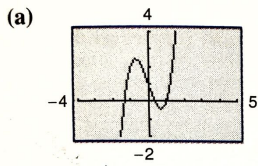
- When $n = \text{ODD}$ the Ends go in **OPPOSITE** directions.

$(+) a_n > 0$ L: as $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$	RISES RIGHT as $x \rightarrow \infty$, $f(x) \rightarrow \infty$.	$(-) a_n < 0$ L: as $x \rightarrow -\infty$, $f(x) \rightarrow \infty$	FALLS RIGHT as $x \rightarrow \infty$, $f(x) \rightarrow -\infty$.
-----------------------------------------------------------------------------	-------------------------------------------------------------------------------	----------------------------------------------------------------------------	--------------------------------------------------------------------------------
- When $n = \text{EVEN}$ the Ends go in the **SAME** direction.

$(+) a_n > 0$ L: as $x \rightarrow -\infty$, $f(x) \rightarrow \infty$	RISES RIGHT as $x \rightarrow \infty$, $f(x) \rightarrow \infty$.	$(-) a_n < 0$ L: as $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$	FALLS RIGHT as $x \rightarrow \infty$, $f(x) \rightarrow -\infty$.
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Math Analysis

Identifying Graphs of Polynomial Functions In Exercises 9–16, match the polynomial function with its graph. [The graphs are labeled (a) through (h).]




10. $f(x) = x^2 - 4x$

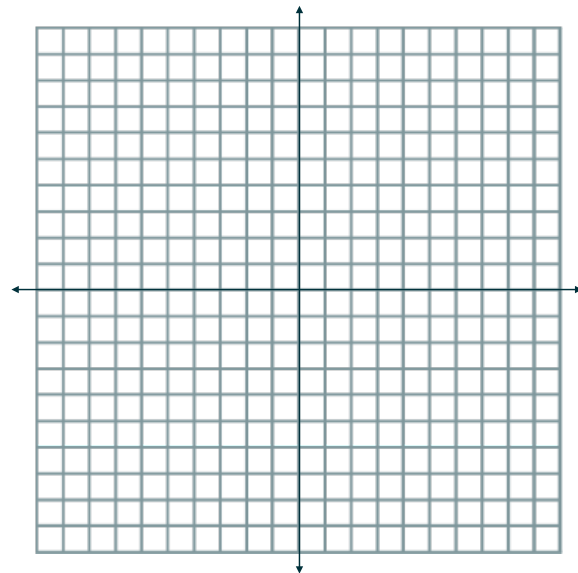
12. $f(x) = 2x^3 - 3x + 1$

14. $f(x) = -\frac{1}{3}x^3 + x^2 - \frac{4}{3}$

16. $f(x) = \frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$

 **Library of Parent Functions** In Exercises 17–22, sketch the graph of $y = x^3$ and the graph of the function f . Describe the transformation from y to f .

20. $f(x) = (x - 2)^3 - 2$



Math Analysis

Comparing End Behavior In Exercises 23–28, use a graphing utility to graph the functions f and g in the same viewing window. Zoom out far enough to see the right-hand and left-hand behavior of each graph. Do the graphs of f and g have the same right-hand and left-hand behavior? Explain why or why not.

24. $f(x) = -\frac{1}{3}(x^3 - 3x + 2)$, $g(x) = -\frac{1}{3}x^3$

28. $f(x) = -(x^4 - 6x^2 - x + 10)$, $g(x) = x^4$

Applying the Leading Coefficient Test In Exercises 29–36, use the Leading Coefficient Test to describe the right-hand and left-hand behavior of the graph of the polynomial function. Use a graphing utility to verify your result.

30. $h(x) = 1 - x^6$

34. $f(x) = \frac{3x^7 - 2x^5 + 5x^3 + 6x^2}{4}$

32. $f(x) = \frac{1}{3}x^3 + 5x$