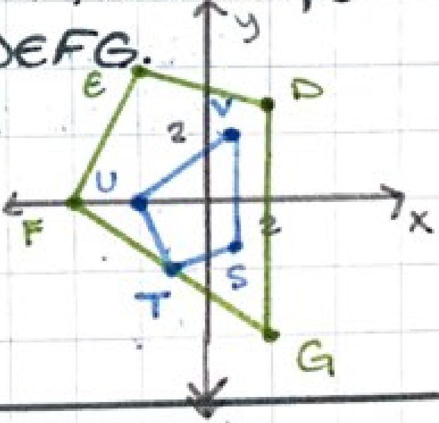
8.3

8.3 Similar Polygons

Objective Today we will use similarity statements, find corresponding lengths in similar polygons, find perimeters & areas of similar polygons, AND we will decide whether polygons are similar.

Warm-up Describe a similarity transformation that maps quadrilateral STUV to quadrilateral DEFG. (From Blue to Green).

8.2 Mini-Assessment #2



Notes:

CORRESPONDING PARTS of SIMILAR POLYGONS

IF $\triangle ABC \sim \triangle DEF$, THEN

$\angle A \cong \angle D$, $\angle B \cong \angle E$, & $\angle C \cong \angle F$

AND $\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA} = k$.

→ Angles are preserved & side lengths are proportional.

CORRESPONDING LENGTHS in SIMILAR POLYGONS

IF two polygons are SIMILAR, then the RATIO of ANY two corresponding lengths in the polygons is EQUAL TO the SCALE Factor of the similar polygons.

PERIMETERS OF SIMILAR POLYGONS THM

IF two polygons are SIMILAR, THEN the Ratio of their Perimeters is equal to the ratio of their corresponding side lengths.

When two SIMILAR polygons have a scale factor of k, the ratio of their perimeters equals k.

AREAS OF SIMILAR POLYGONS THM

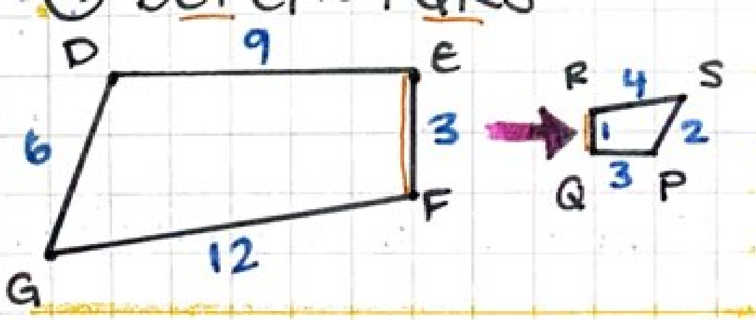
IF two polygons are SIMILAR, THEN the Ratio of their Areas is equal to the squares of the ratios of their corresponding side lengths.

When two SIMILAR polygons have a scale factor of k, the ratio of their areas is equal to k^2.

Practice Problems

Find the scale factor. Then list ALL pairs of congruent angles & write the ratios of the corresponding side lengths in a statement of proportionality.

(4) DEFG ~ PQRS



$k = \frac{1}{3}$

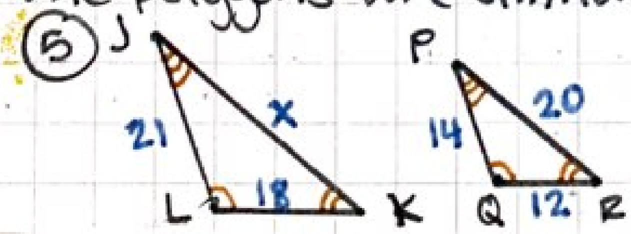
$\angle D \cong \angle P$, $\angle E \cong \angle Q$,

$\angle F \cong \angle R$, $\angle G \cong \angle S$,

$\frac{PQ}{DE} = \frac{QR}{EF} = \frac{RS}{FG} = \frac{SP}{GD}$

The polygons are similar. Find the value of x.

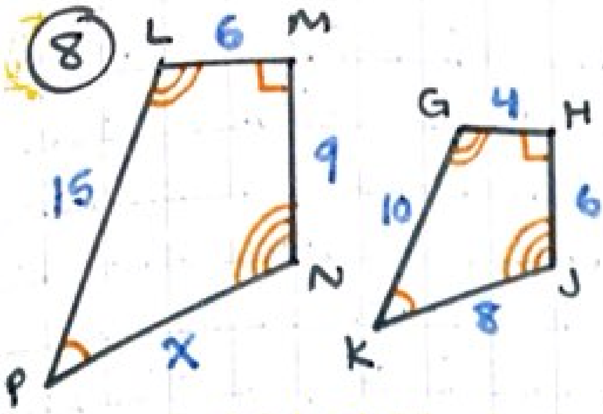
(5)



$\frac{x}{20} = \frac{18}{12} \rightarrow \frac{3}{2}$ $\quad 2x = 60$

$x = 30$

(8)



$\frac{x}{8} = \frac{9}{6} \Rightarrow \frac{3}{2}$

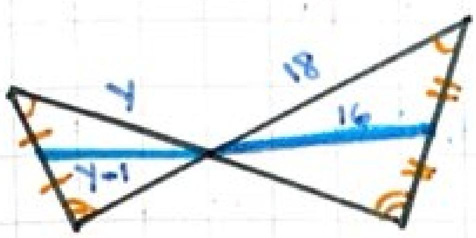
$\Rightarrow 2x = 24$

$x = 12$

The black triangles are similar. Identify the type of segment shown in blue & find the value of the variable.

↱ median, Altitude, ⊥ bisector, ∠ bisector (these are proportional in similar Δs too).

(10)



vertex → opp. midpoint

⇒ Median

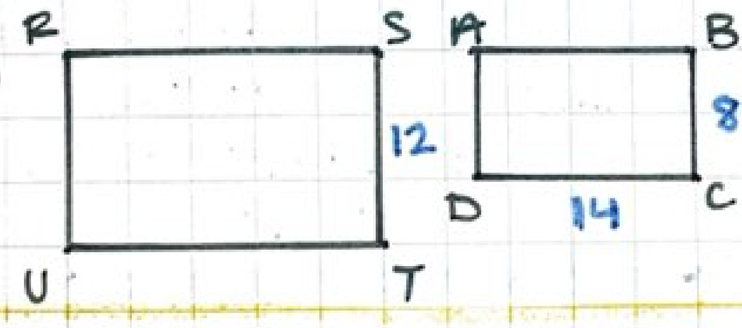
$\frac{y}{18} = \frac{y-1}{16}$

$16y = 18y - 18$

$18 = 2y \Rightarrow y = 9$

RSTU ~ ABCD. Find the ratio of their perimeters:

(11)



$K = \frac{8}{12} = \frac{2}{3}$

Perimeter Ratio: 2:3

Two polygons are similar. The perimeter of one polygon & the ratio of the corresponding side lengths are given. Find the perimeter of the other polygon.

↱ ratio of Perim = k

(13) Perim. of smaller: 48 cm ; ratio: $\frac{2}{3}$

$\frac{\text{P of small}}{\text{P of big}} = \text{ratio} \Rightarrow \frac{48\text{ cm}}{x} = \frac{2}{3}$

$\Rightarrow 2x = 144 \Rightarrow x = 72 \Rightarrow 72\text{cm}$

(16) Perimeter of larger: 85 m ; ratio: $2/5$

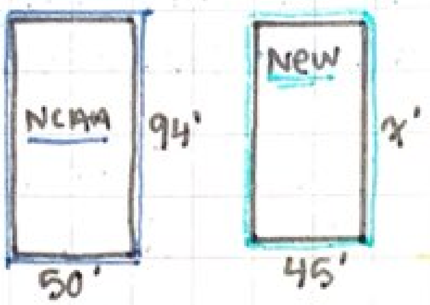
$x = \frac{\text{Perim. of Smaller}}{85\text{ m}} = \frac{2}{5} \Rightarrow 5x = 170$

$x = 34$

34m = Perim. of smaller

(17) A school gymnasium is being remodeled. The basketball court will be similar to an NCAA basketball court, which has a length of 94 feet & width of 50 feet. The school plans to make the width of the new court 45 feet.

Find the perimeters of the NCAA court and of the new court in the school.



NCAA Perimeter = 288 ft.

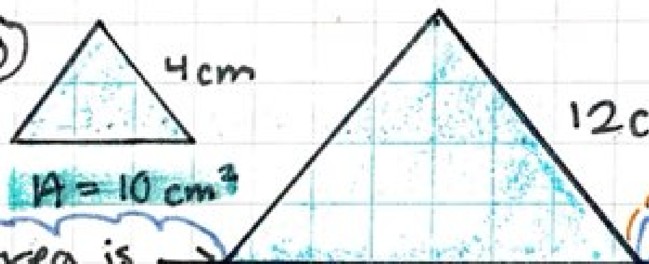
$K = \frac{45}{50} = \frac{9}{10} \Rightarrow x = 84.6\text{ ft}.$

New Perimeter $= 288\left(\frac{9}{10}\right)$
$= 259.2\text{ ft}.$

Ratio of areas $= k^2$

The polygons are similar. The AREA of one polygon is given. **Find the area of the other polygon.**

(20)



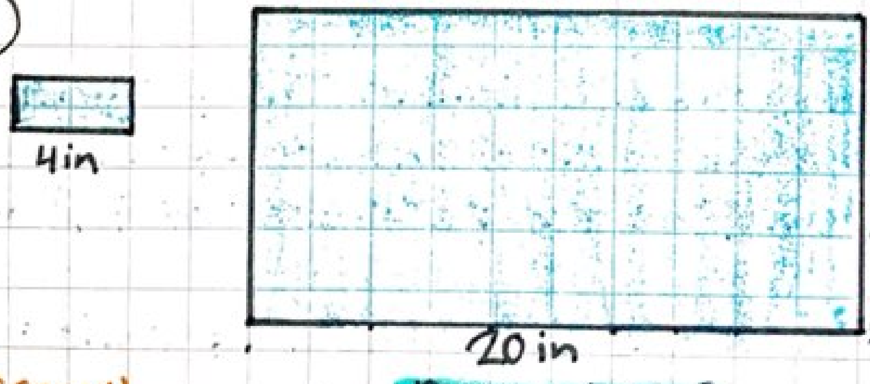
$A = 10\text{ cm}^2$

Area is $(k)^2$ · Area of smaller

$\frac{4\text{ cm}}{12\text{ cm}} = K \Rightarrow \boxed{K = \frac{1}{3}}$

Area of Big $= 3^2$(area of small)
$= 90\text{ cm}^2$

(21)

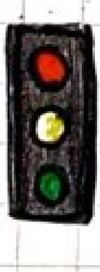


Area $= 100\text{ in}^2$

$\boxed{K = \frac{4}{20} = \frac{1}{5}}$

A of Small
$= \left(\frac{1}{5}\right)^2$ A of Big
$= \frac{1}{25}(100) = 4 \rightarrow$ Area of Small $= 4\text{ in}^2$

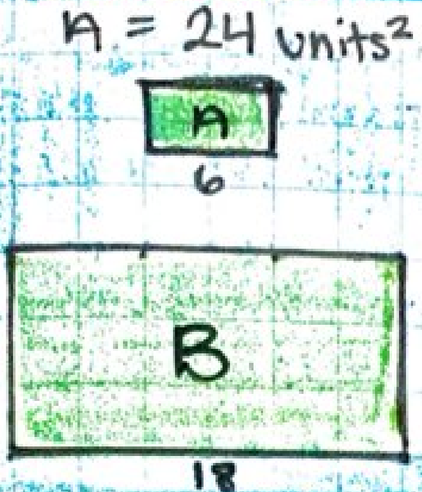
HW 8.3a
(Pg. 481)
#3, 6, 7, 9, 12, 14, 15, 18, 19, 22, 23, 26, 27



8.3 Day 2

Warmup

Describe & Correct the ERROR in finding the area of rectangle B. The rectangles are similar



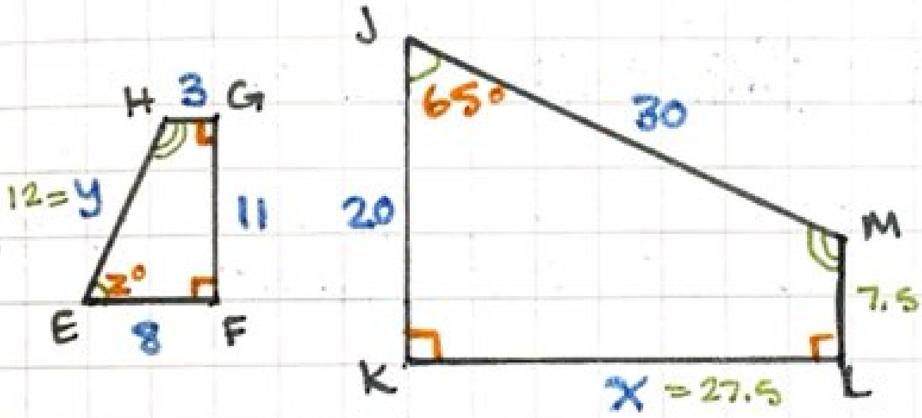
$\frac{6}{18} = \frac{24}{x}$

$6x = 432$

$x = 72$

Notes/ Practice Problems

For #28-34 JKLM ~ EFGH.



$z = 65°$

$y = \frac{2}{5}(30) = 12$

$\frac{11}{x} = \frac{2}{5} \Rightarrow x = \frac{5}{2}(11)$

$x = 55/2 = 27.5$

$ML = \frac{5}{2}(3) = \frac{15}{2} = 7.5$

(29) Find the scale factor of EFGH to JKLM.

$k = \frac{EF}{JK} = \frac{8}{20} = \frac{2}{5} = k$

(31) Find the Perimeter of Each Polygon.

P of EFGH = 34 units

P of JKLM = 85 units

(33) Find the Area of Each Polygon.

Area of EFGH = 33 + 27.5 = 60.5 units²

↳ □ + △ ↗

Area of JKLM = □ + ▷ = 206.25 + 171.875

= 378.125 units²

The figures are similar. Find the missing corresponding side length.

(40) Fig. A has a Perim. of 24". Fig. B has a Perim. of 36" & one side is 12".

$\frac{2}{3} = \frac{24}{36} = \frac{x}{12} \Rightarrow 3x = 24$

$x = 8$ in.

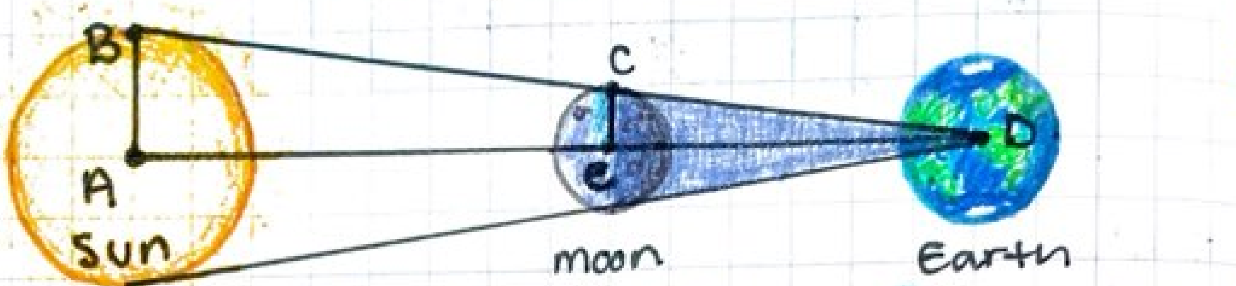
(41) Fig. A has an Area of 48 sq. ft. & one side of 6 ft. Fig. B has an area of 75 ft².

$\frac{16}{25} = \frac{48}{75} = \left(\frac{6}{x}\right)^2 \Rightarrow \frac{4}{5} = \frac{6}{x} \Rightarrow 4x = 30$

$x = 7.5$ ft

(51) During a total eclipse of the Sun, the moon is directly in line with the Sun & blocks the Sun's rays. The distance DA between Earth and the Sun is 93,000,000 miles, the distance DE between the Earth and moon is 240,000 miles, and the radius AB of the Sun is 432,500 miles.

Use the diagram & given measurements to estimate the radius EC of the moon.

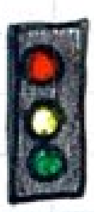


$\triangle DEC \sim \triangle DAB$ (Not drawn to scale)

$$K = \frac{DE}{DA} = \frac{240K}{93000K} = \frac{24}{9300} = \frac{6}{2325} = K$$

$$\frac{DE}{DA} = \frac{EC}{AB} \Rightarrow \frac{6}{2325} = \frac{EC}{432500} \Rightarrow EC \approx 1116.129 \text{ mi}$$

HW 8.3b (Pg. 481) #28-34 even, 37, 39, 42-48, 50, 55



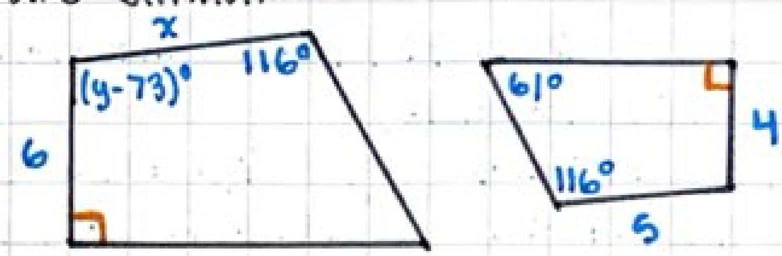
8.4

8.4 PROVING △ Similarity by AA

Objective: Today we will use the Angle-Angle Similarity Theorem and we will solve real-life problems.

Warm-up: Find the values of x and y when the two polygons are similar.

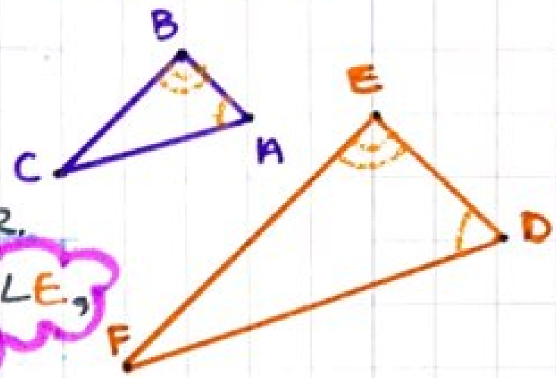
8.3 #38 →



Notes

ANGLE-ANGLE (AA) SIMILARITY THM

IF two angles of one $\triangle$ are CONGRUENT to two angles of another $\triangle$, THEN the two triangles are SIMILAR.



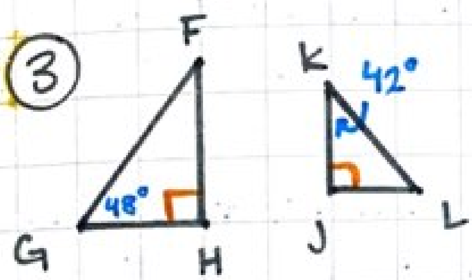
IF $\angle A \cong \angle D$ and $\angle B \cong \angle E$,

→ THEN $\triangle ABC \sim \triangle DEF$.

Practice Problems

Determine whether the triangles are similar. If they are, write a similarity statement. Explain.

(3)

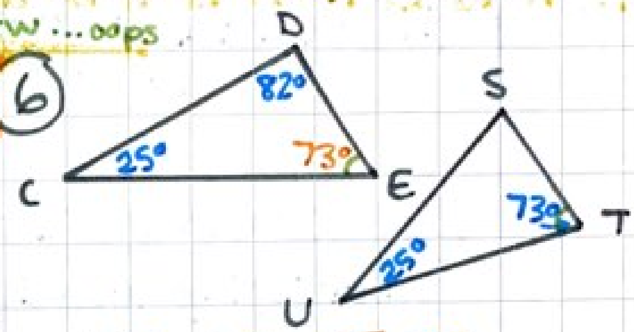


$m\angle F = 90 - 48$
$= 42°$

Yes! $\angle F \cong \angle K$ & $\angle J \cong \angle H$
So $\triangle FGH \sim \triangle KLJ$.

HW... oops

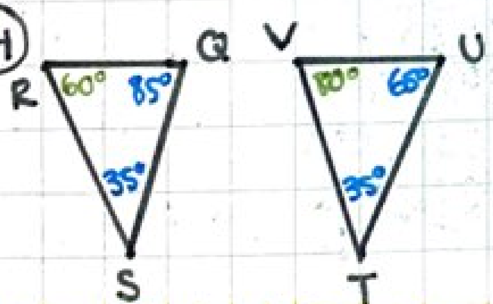
(6)



$m\angle E = 180 - 25 - 82$
$= 73° \cong m\angle T$

Yes! $\angle C \cong \angle U$ & $\angle E \cong \angle T$
So $\triangle CDE \sim \triangle UST$.

(4)

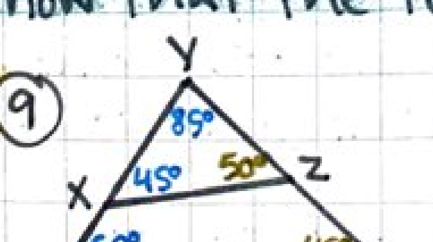


$m\angle R = 180° - 35° - 85°$
$= 60° \neq m\angle U$

No! $m\angle R = 60° \neq m\angle U$
& $m\angle Q \neq m\angle V$.

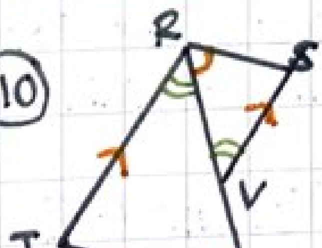
Show that the two triangles are similar.

(9)



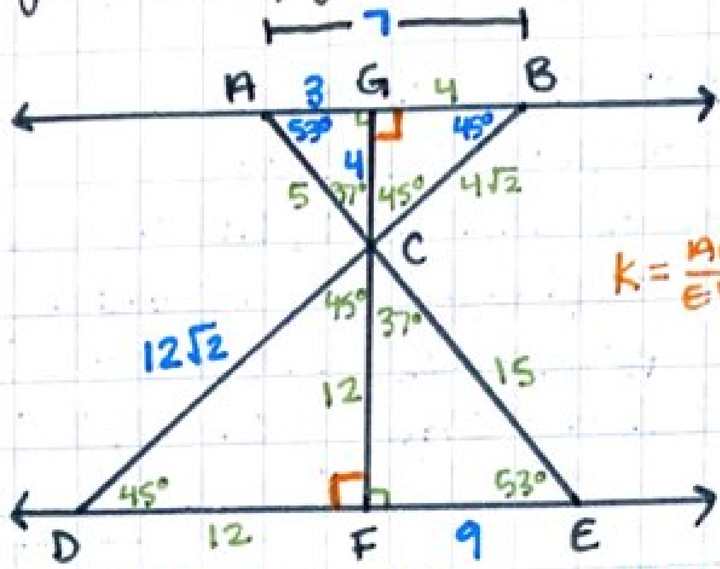
① $\angle Y \cong \angle Y$ (Refl. Prop.)
$m\angle XZY = 50° \Rightarrow$
② $\angle XZY \cong \angle W$
So $\triangle XYZ \sim \triangle UYW$.

(10)



① $\angle U \cong \angle VRS$ (Given)
② $\angle TRU \cong \angle SVR$ (Alt. Int. ∠'s Thm)
So $\triangle TRU \sim \triangle SVR$.

Use the diagram to copy and complete the statement.



$k = \frac{AG}{EF} = \frac{3}{9} = \frac{1}{3} = k$

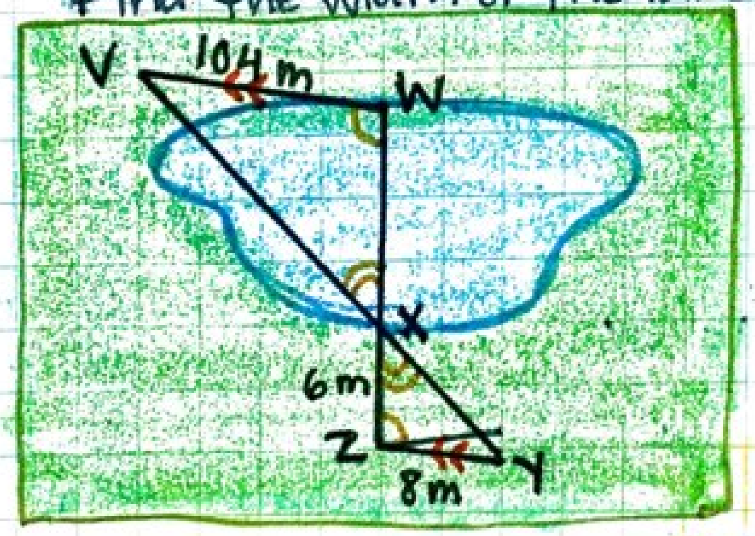
(12) $\triangle DCF \sim \triangle BCG$

(16) $CF = 12 = 3GC = 3(4)$ OR $= DF$

(15) $m\angle ECD = 82°$ $= 45+37 =$

(17) $BC = 4\sqrt{2}$ $= \frac{1}{3}(12\sqrt{2})$

(21) You can measure the width of ~~the~~ lake using a surveying technique, as shown in the diagram. Find the width of the lake, WX. Justify your answer.



$K = \frac{8}{104}$

OR

$\frac{8}{104} = \frac{6}{WX}$

$\Rightarrow WX = 78m$

$\angle W \cong \angle Z$ by Alt. Int. ∠'s Thm, $\angle WXV \cong \angle ZXY$ by Vert. ∠'s Thm, So $\triangle WXV \sim \triangle ZXY$

HW 8.4 (Pg. 489) #5-8, 11, 13, 14, 18, 20, 27, 28, 33

Stoplight Reflection

Review your Notes & HW and categorize what you know, what you think you know, & what you do not know.

● What do I need to Relearn/STUDY FIRST? Write down questions for 5-min Review.

● What do I THINK I KNOW, BUT still have a couple questions about?

● What I KNOW & Could teach to someone else.

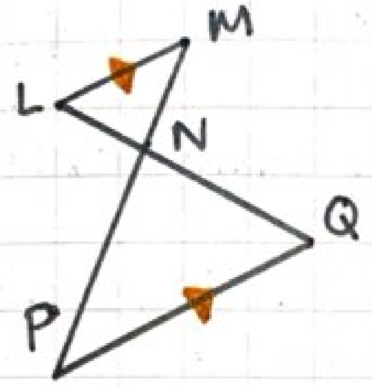
8.5

8.5 PROVING Δ SIMILARITY by SSS and SAS

Objective Today we will use the Side-Side-Side Similarity Thm and we will use the Side-Angle-Side Similarity Thm.

Warm-up Show that $\triangle LMN \sim \triangle QPN$ WITHOUT using Vertical Angles. (Write as a two-column Proof.)

8.4 ↗ Mini-Assmt. #2

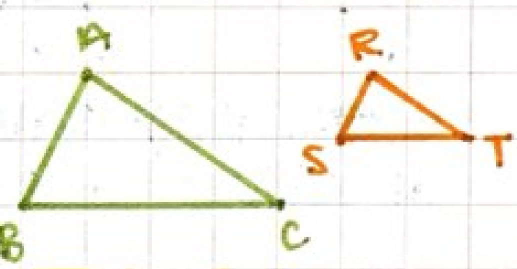


Statements	Reasons
1.	1. Given
2.	2.
3. $\triangle LMN \sim \triangle QPN$	3.

Notes

SIDE-SIDE-SIDE (SSS) SIMILARITY THM

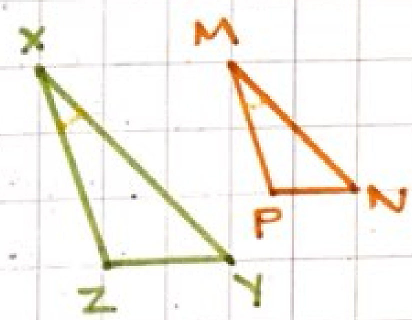
IF the corresponding side lengths of two triangles are PROPORTIONAL, THEN the triangles are SIMILAR.



IF $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, THEN $\triangle ABC \sim \triangle RST$.

SIDE-ANGLE-SIDE (SAS) SIMILARITY THM

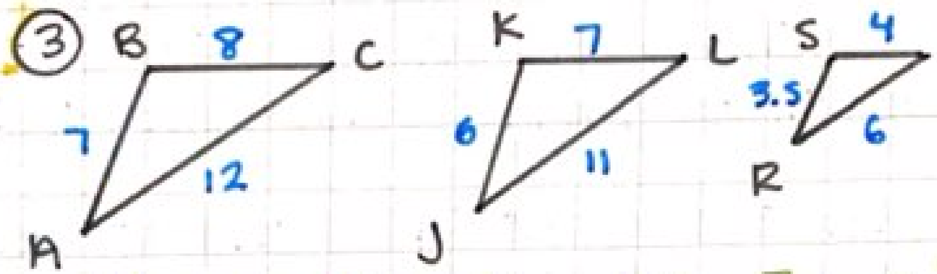
IF an Angle of one Δ is congruent to an Angle of a second Δ AND the lengths of the sides INcluding these Angles are Proportional, THEN the triangles are SIMILAR.



IF $\angle X \cong \angle M$ AND $\frac{ZX}{PM} = \frac{XY}{MN}$, THEN $\triangle XYZ \sim \triangle MNP$.

Practice Problems

Determine whether $\triangle JKL$ or $\triangle RST$ is similar to $\triangle ABC$.

③ 

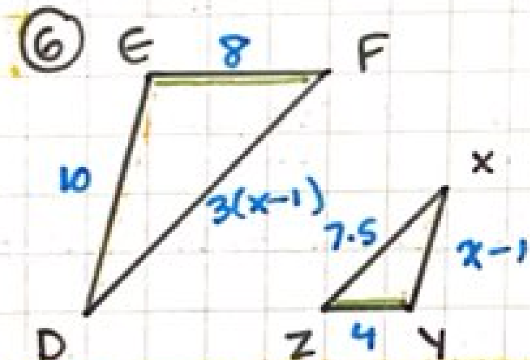
$\triangle RST$

$\frac{AB}{JK} = \frac{7}{6} \neq \frac{BC}{KL} = \frac{8}{7}$

$\frac{AB}{RS} = \frac{7}{3.5} = 2 \overset{\checkmark}{=} \frac{BC}{ST} = \frac{8}{4} = 2$

$\overset{\checkmark}{=} \frac{CA}{TR} = \frac{12}{6} = 2$

Find the value of x that makes $\triangle DEF \sim \triangle XYZ$.

⑥ 

$\frac{EF}{YZ} = \frac{8}{4} = 2 = \frac{DE}{XY}$

$\Rightarrow 2XY = DE$

$2(x-1) = 10$

$x-1 = 5$

$x = 6$

Verify that $\triangle ABC \sim \triangle DEF$. Find the scale factor of $\triangle ABC$ to $\triangle DEF$. $\Rightarrow \frac{\triangle DEF}{\triangle ABC}$

⑦ $\triangle ABC$: $BC = 18$, $AB = 15$, $AC = 12$

$\triangle DEF$: $EF = 12$, $DE = 10$, $DF = 8$

$\frac{AB}{DE} = \frac{15}{10} = \frac{3}{2}$ $\frac{BC}{EF} = \frac{18}{12} = \frac{3}{2}$ $\frac{AC}{DF} = \frac{12}{8} = \frac{3}{2}$

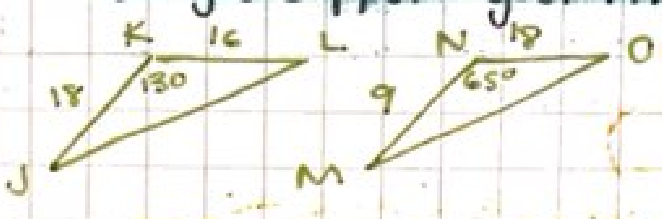
$K = \frac{DE}{AB}$

$\Downarrow$

$K = \frac{2}{3}$

⑮ Your friend claims that $\triangle JKL \sim \triangle MNO$ by the SAS Similarity Thm when $JK = 18$, $m\angle K = 130°$, $KL = 16$, $MN = 9$, $m\angle N = 65°$, & $NO = 18$.

Do you support your friend's claim? Explain.



No! If $\triangle JKL \sim \triangle MNO$, then $\angle K \cong \angle N$, BUT $\angle K \ncong \angle N$.

also $\frac{JK}{MN} \neq \frac{KL}{NO} \Rightarrow \triangle JKL \nsim \triangle MNO$.

HW 8.5
(Pg. 497)
#1, 4, 5, 8, 9, 14, 19, 20

8.6 Vocab.	1. DIRECTED LINE SEGMENT: a DIRECTED LINE SEGMENT $\overline{AB}$ is a segment that represents moving from point A to point B.

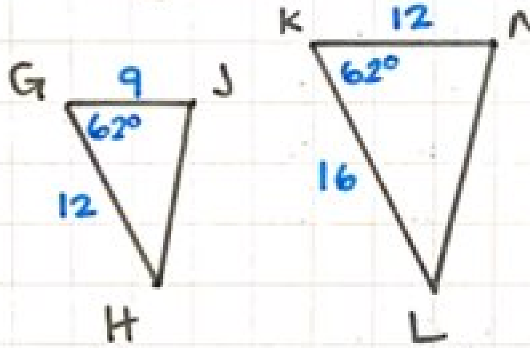
8.6

8.6 PROPORTIONALITY THEOREMS

Objective: Today we will use proportionality theorems and partition directed line segments.

Warmup (8.5 Mini-Assess. #3)

Determine whether the triangles are similar. IF they are, FIND the scalefactor of $\triangle GHJ$ to $\triangle KLM$.



Notes:

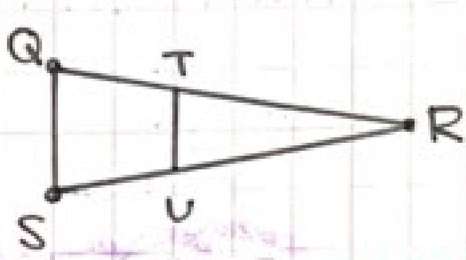
TRIANGLE PROPORTIONALITY THM

IF a line PARALLEL to one side of a $\triangle$ intersects the other two sides, THEN it divides the two sides PROPORTIONALLY.

IF $\overline{TU} \parallel \overline{QS}$, THEN $\frac{RT}{TQ} = \frac{RU}{US}$.

CONVERSE OF THE $\triangle$ PROPORTIONALITY THM

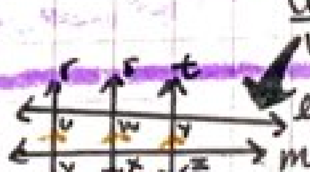
IF a line divides two sides of a $\triangle$ proportionally, THEN it is PARALLEL to the third side.



IF $\frac{RT}{TQ} = \frac{RU}{US}$, THEN $\overline{TU} \parallel \overline{QS}$.

THREE PARALLEL LINES THM

$\frac{UW}{WY} = \frac{VX}{XZ}$



IF 3 Parallel lines intersect 2 transversals, THEN they divide the transversals proportionally.

TRIANGLE ANGLE BISECTOR THM

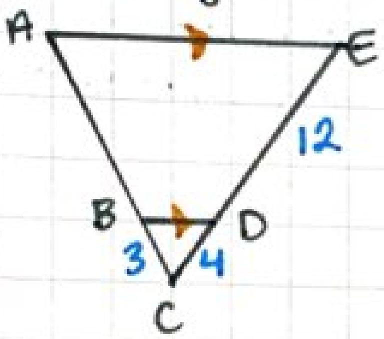
IF a RAY Bisects ANY Angle of a triangle, THEN it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

$$\frac{AD}{DB} = \frac{CA}{CB}$$

Practice Problems

Find the length of $\overline{AB}$.

(3)



Since $\overline{AE} \parallel \overline{BD}$,

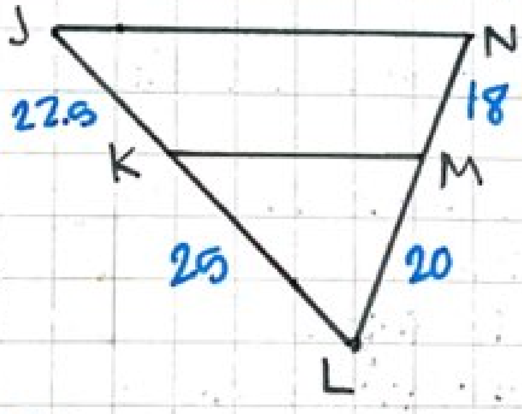
$$\frac{AB}{3} = \frac{12}{4}$$

$$4AB = 36$$

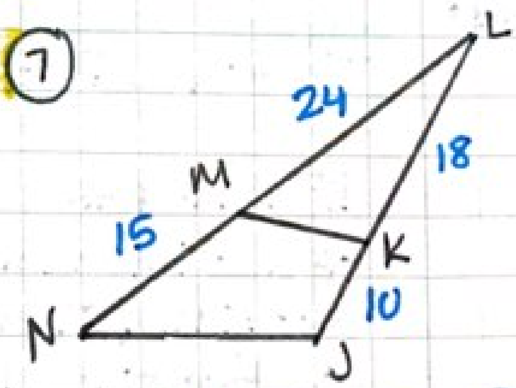
$$AB = 9$$

Determine whether $\overline{KM} \parallel \overline{JN}$.

(6)



(7)



$\frac{22.5}{25} = \frac{9}{10}$ $\quad \frac{18}{20} = \frac{9}{10}$

$\Rightarrow \frac{JK}{KL} = \frac{NM}{ML} \Rightarrow$ yes $\overline{KM} \parallel \overline{JN}$.

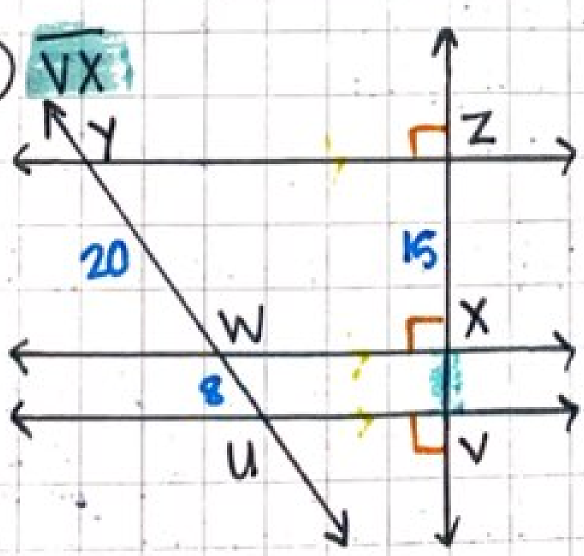
$\frac{24}{15} = \frac{8}{5}$ $\quad \frac{18}{10} = \frac{9}{5}$

$\Rightarrow$ NO!

$\Rightarrow \overline{KM} \nparallel \overline{JN}$.

Find the length of the indicated line segment.

(11) $\overline{VX}$



By Lines $\perp$ to a Transversal Thm $\overline{YZ} \parallel \overline{WX} \parallel \overline{UV}$.

$$\Rightarrow \frac{YW}{WU} = \frac{ZX}{XV}$$

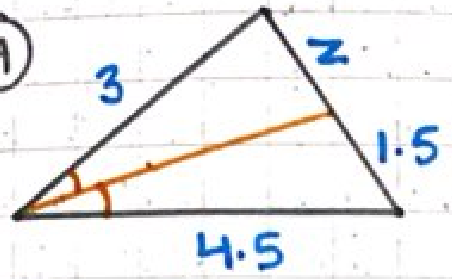
$$\Rightarrow \frac{20}{8} = \frac{15}{VX}$$

$$\Rightarrow 20\,VX = 120$$

$$VX = 6$$

Find the value of the variable.

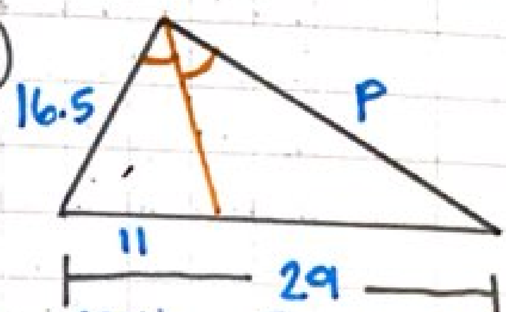
(14)



$\frac{z}{1.5} = \frac{3}{4.5} \Rightarrow 4.5z = 3(1.5)$

$z = 1$

(15)

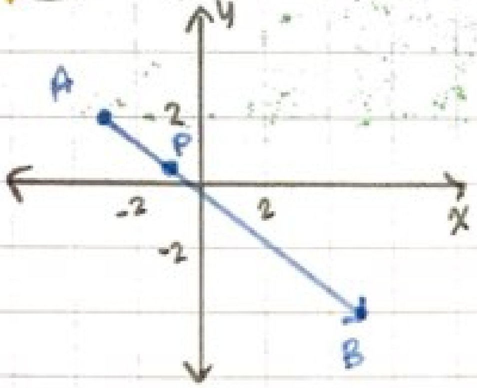


$\frac{29-11}{11} = \frac{p}{16.5} \Rightarrow 18(16.5) = 11p$

$p = \frac{297}{11} \Rightarrow p = 27$

Find the coordinates of point P along the directed line segment AB so that AP & PB is the given ratio.

(20) $A(-3,2), B(5,-4)$; 2 to 6



① Divide AB into $2+6=8$ segments.
$\Rightarrow$ P is $\frac{2}{8} = \frac{1}{4}$ away from A.
② ADD $\frac{1}{4}$ of RUN to x-coordinate of A
ADD $\frac{1}{4}$ of RISE to y-coordinate of A

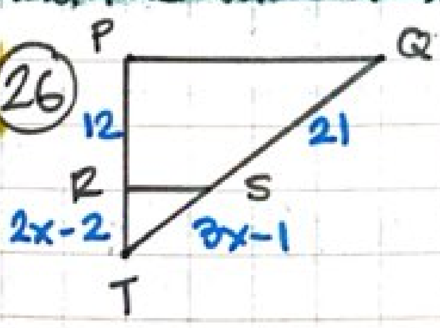
$m_{AB} = \frac{-6}{8} = \frac{RISE}{RUN} \Rightarrow P_x = (-3 + \frac{1}{4}(8)) = -1$

$\Rightarrow P_y = (2 + \frac{1}{4}(-6)) = \frac{1}{2}$

$P(-1, \frac{1}{2})$

Find the value of x for which $\overline{PQ} \parallel \overline{RS}$.

(26)



$\frac{2x-2}{12} = \frac{3x-1}{21} \Rightarrow 42x - 42 = 36x - 12$

$-30 = -6x$

$5 = x$

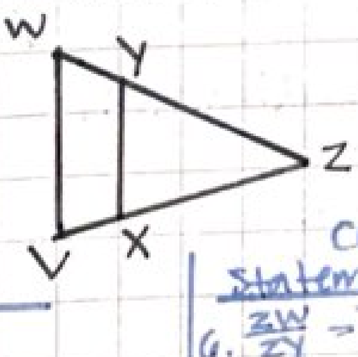
$\Rightarrow x = 5$

Prove the CONVERSE of the TRIANGLE PROPORTIONALITY THM.

(28)

Given: $\frac{ZY}{YW} = \frac{ZX}{XV}$

Prove: $\overline{YX} \parallel \overline{WV}$

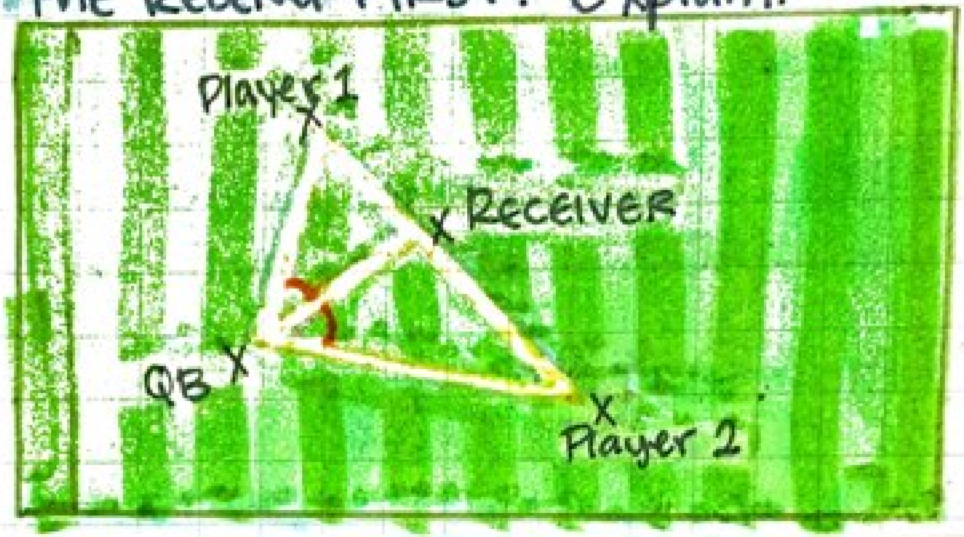


Statements	Reasons
1. $\frac{ZY}{YW} = \frac{ZX}{XV}$	1. Given
2. $\frac{YW}{ZY} = \frac{XV}{ZX}$	2. Rewriting Proportion
3. $\frac{YW}{ZY} + \frac{ZY}{ZY} = \frac{XV}{ZX} + \frac{ZX}{ZX}$	3. Add. Prop.
4. $\frac{YW+ZY}{ZY} = \frac{XV+ZX}{ZX}$	4. Simplifying
5. $ZW = ZY + YW$, $ZV = ZX + XV$	5. Seg. Add. Post.

continued.

Statements	Reasons
6. $\frac{ZW}{ZY} = \frac{ZV}{ZX}$	6. Subst. Prop
7. $\angle Z \cong \angle Z$	7. Refl. Prop.
8. $\triangle ZWV \sim \triangle ZYX$	8. SAS Similarity
9. $\angle ZYX \cong \angle ZWV$	9. CASTC
10. $\overline{YX} \parallel \overline{WV}$	10. Corr. ∠s Converse

(34) During a football game, the quarterback throws the ball to the receiver. The receiver is between two defensive players, as shown. If Player 1 is closer to the quarterback when the ball is thrown & both defensive players move at the same speed, which player will reach the Receiver FIRST? Explain.



Player 1, since the angle is bisected, the lengths are proportional.

HW 8.6 (Pg. 504) #4, 5, 8, 9, 12, 13, 16, 18, 24, 25, 27, 30, 33, 37, 38

9.1 Vocab.

1: PYTHAGOREAN TRIPLE: a PYTHAGOREAN TRIPLE is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$.