END SEM. 1

0.0

1.1 Absolute Value Functions

Objective: Today we will Translate, stretch, shrink, & reflect graphs of Absolute value functions. We will also combine transformations of graphs of absolute value functions.

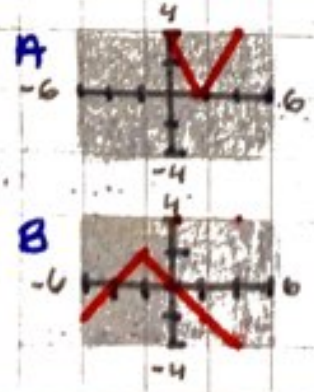
Warmup

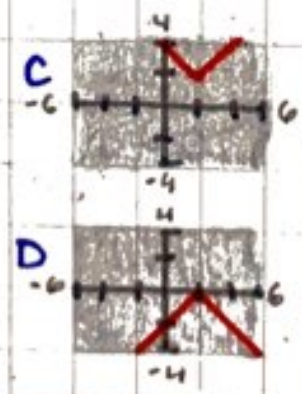
Match each absolute value function with its graph.

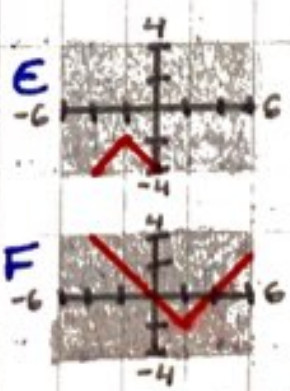
Explor. #1

ⓐ $g(x) = -|x-2|$ ⓑ $g(x) = |x-2|+2$ ⓒ $g(x) = -|x+2|-2$

ⓓ $g(x) = |x-2| \div 2$ ⓔ $g(x) = 2|x-2|$ Ⓕ $g(x) = -|x+2|+2$







Notes

- An **ABSOLUTE VALUE FUNCTION** is a function that contains an absolute value expression, and its graph is V-shaped and symmetric (through its vertex).
  - The **VERTEX** is the point where the graph changes direction.

PARENT ABSOLUTE VALUE FUNCTION

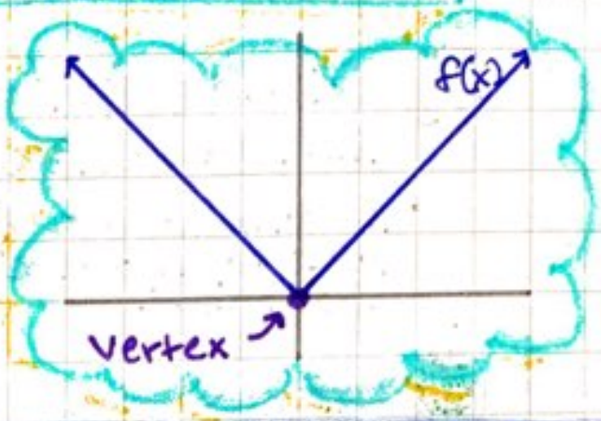
$f(x) = |x|$

Vertex: (0,0)

Domain: $\mathbb{R}$ = all Real #s

Range: $y \geq 0$

Opens Up ↑



VERTEX FORM of an ABSOLUTE VALUE FUNCTION

Given the vertex (h,k) of an absolute value function, its function written in **VERTEX FORM** is

$$g(x) = a|x-h|+k \quad \text{where } a \neq 0.$$

"a" determines the slopes AND whether it opens up or down
- $a > 0$ ⇒ open up
- $a < 0$ ⇒ Open Down

* its graph is SYMMETRIC about the line $x = h$.

STEPS FOR GRAPHING

1. Identify the vertex (h,k)
2. Make a table of values
   (Hint: INCLUDE the vertex... pick values around h.)
3. PLOT the ordered pairs
4. DRAW the V-shaped graph

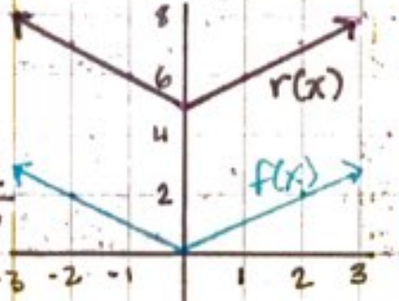
Practice Problems

Graph the function. Compare it to the graph of $f(x)=|x|$. Describe the domain and range.

⑥ $r(x) = |x|+5$

V: (0,5)

x	-2	-1	0	1	2
r(x)	7	6	5	6	7



r(x) is translated 5 units up from f(x)

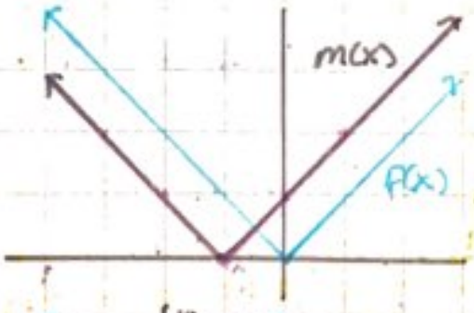
D: $\mathbb{R}$

R: $y \geq 5$

(7) $m(x) = |x+1|$

V: (-1, 0)

x	-3	-2	-1	0	1
m(x)	2	1	0	1	2



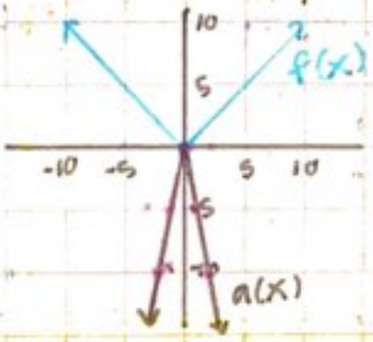
m(x) is shifted 1 unit to the Left from f(x).

D: ℝ R: $y \geq 0$

(11) $a(x) = -5|x|$

V: (0, 0)

x	-2	-1	0	1	2
a(x)	-10	-5	0	-5	-10



a(x) has a Vert. Stretch by a factor of 5 & a Vert. Reflection in the x-axis.

D: ℝ R: $y \leq 0$

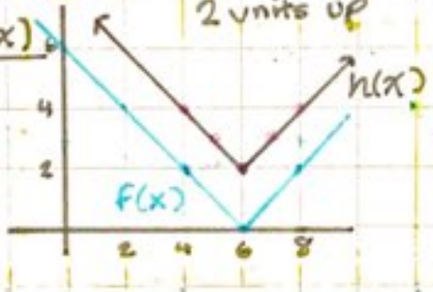
Graph the function. Compare it to the graph of $f(x) = |x-6|$.

(13) $h(x) = |x-6|+2$

V: (6, 2)

h(x) is trans. 2 units up

x	h(x)
4	4
5	3
6	2
7	3
8	4

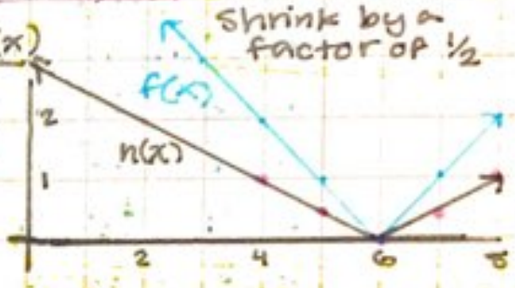


(14) $n(x) = \frac{1}{2}|x-6|$

V: (6, 0)

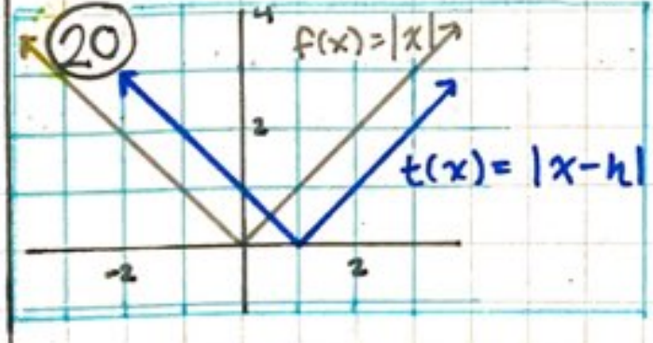
n(x) is a vert. shrink by a factor of ½

x	n(x)
4	1
5	½
6	0
7	½
8	1



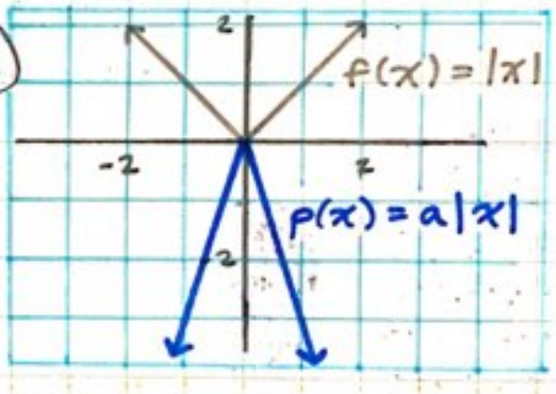
Compare the graphs, Find the value of h, k, or a.

(20)



t(x) is translated 1 unit to the right. $h = 1$

(21)



a(x) is reflected in the x-axis & has a Vert. Stretch by a factor of 3. $a = -3$

Write an equation that represents the given transformation(s) of $g(x) = |x|$.

(23) Vertical translation 7 units down.

$h(x) = |x| - 7$

(26) Vertical stretch by a factor of 3 & a reflection in the x-axis.

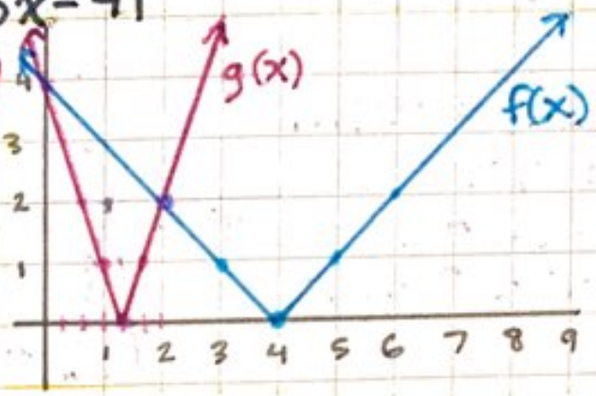
$h(x) = -3|x|$

Graph & compare the two functions.

(27) • $f(x) = |x-4|$
• $g(x) = |3x-4|$

x	f(x)
2	2
3	1
4	0
5	1
6	2

x	g(x)
2/3	2
1	1
4/3	0
5/3	1
2	2

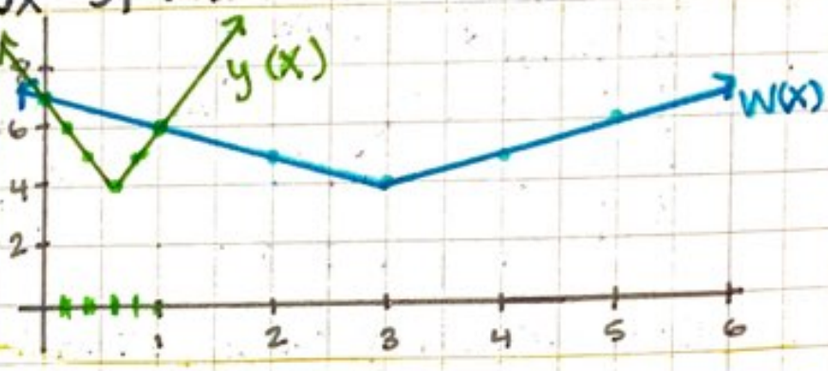


↑ x-values, 1/3 of x-values for f(x) for the same y-values. ⇒ g(x) is a HORIZ. shrink of f(x) by a factor of 1/3

(30) • $w(x) = |x-3| + 4$
• $y(x) = |5x-3| + 4$

x	w(x)
1	6
2	5
3	4
4	5
5	6

x	y(x)
1/5	6
2/5	5
3/5	4
4/5	5
1	6



↑ x-values, 1/5 of the x-values for w(x) for the same y-values ⇒ y(x) is a Horiz. shrink of w(x) by a factor of 1/5

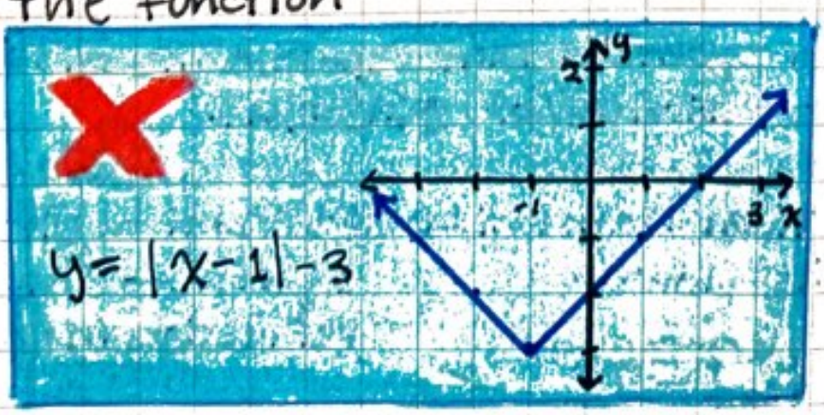
HW 1.1 A
(Pg. 8)
2, 5, 8, 12, 15, 16, 19, 22, 24, 25, 28, 31

1.1 Day 2

Warm-up

↑ #45.

Describe & CORRECT the ERROR in graphing the function



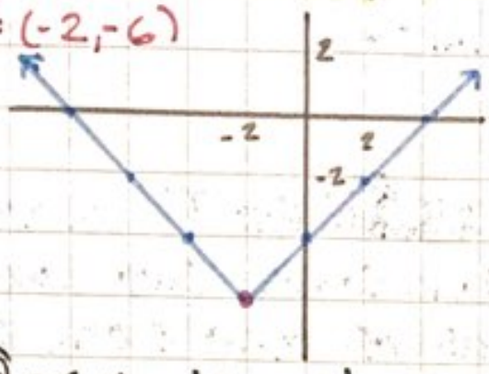
Notes/ Practice Problems

Describe the transformations from the graph of $f(x)=|x|$ to the graph of the given function. Then GRAPH the given function.

(33) $r(x)=|x+2|-6$

Translated 2 units to the left & 6 units down

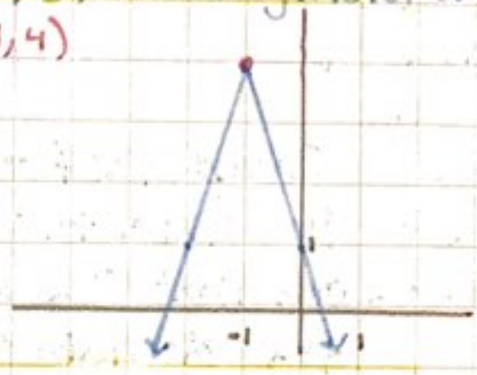
V: (-2,-6)



(36) $v(x)=-3|x+1|+4$

1 unit Left, 4 units up, reflection in x-axis & vert. stretch by factor of 3.

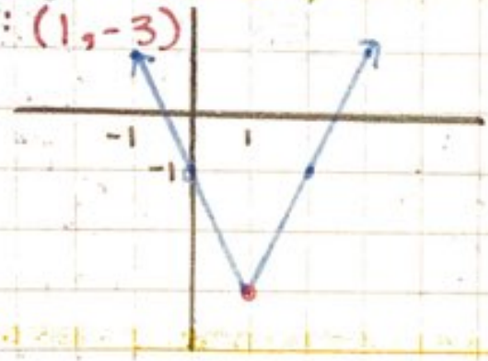
V: (-1,4)



(38) $s(x)=|2x-2|-3$

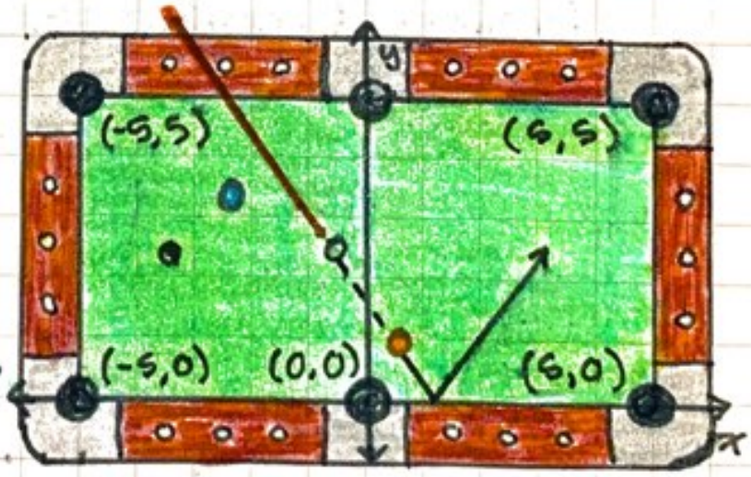
2 Right, 3 down, & Horiz. shrink by factor of 1/2

V: (1,-3)



(42) On the pool table shown, you bank the five ball off the side represented by the x-axis. The path of the ball is described by the function

$p(x)=\frac{4}{3}\left|x-\frac{5}{4}\right|$.



(a) At what point does the five ball bank off the side?

The vertex: $\left(\frac{5}{4},0\right)$

(b) Do you make the shot? EXPLAIN.

Yes

$\frac{4}{3}\left|5-\frac{5}{4}\right| \stackrel{?}{=} 5$

$\frac{4}{3}\left|\frac{20}{4}-\frac{5}{4}\right| = \frac{4}{3}\left|\frac{15}{4}\right| \rightarrow \frac{15}{3}=5 \stackrel{\checkmark}{=} 5$

(44) Explain how the graph of each function compares to the graph of $y=|x|$ for positive & negative values of k, h, & a.

(a) $y=|x|+k$ — $+k \Rightarrow$ up; $-k \Rightarrow$ down

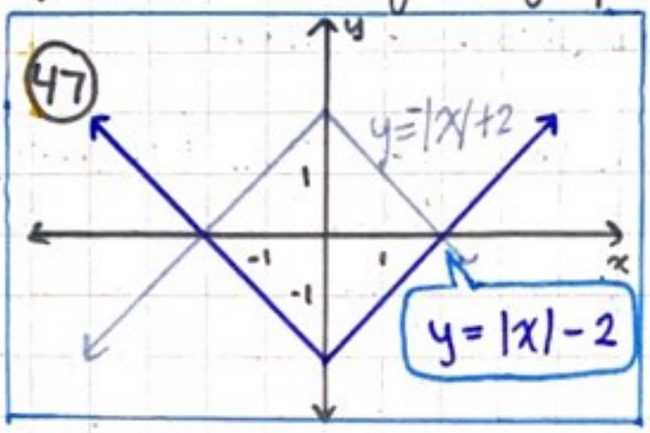
(b) $y=|x-h|$ — $+h \Rightarrow$ Right; $-h \Rightarrow$ Left

(c) $y=a|x|$ — $+a \rightarrow$ vert. stretch/shrink; $-a \rightarrow$ refl. in x-axis

(d) $y=|ax|$ — $|a| \Rightarrow$ Horiz. stretch/shrink; $-a \Rightarrow$ reflect. y-axis

Write an absolute value function whose graph forms a square with the given graph.

(47)



$y = -|x| + 2$

or

$y = -|x| +$ any $k > -2$

h is the same, k needs to change & a needs to become -a.

Graph the function. ~~Describe~~ Then REWRITE the Absolute Value Function as TWO LINEAR functions one that has a domain of $x<0$ & one that has the domain $x \geq 0$.

(52) $y = |x| - 3$

V: (0, -3)

m = -1, b = -3

m = 1, b = -3

$y = -x - 3$ for $x < 0$

$y = x - 3$ for $x \geq 0$

(53) $y = -|x| + 9$

V: (0, 9)

m = +1, b = 9

m = -1, b = 9

$y = x + 9$, $x < 0$

$y = -x + 9$ for $x \geq 0$

Graph and Compare the two functions.

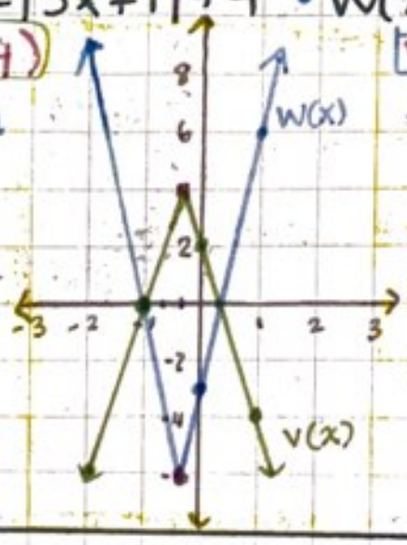
(57) $v(x) = -2|3x+1| + 4$ • $w(x) = 3|3x+1| - 6$

V: $(-\frac{1}{3}, 4)$

V: $(-\frac{1}{3}, -6)$

x	v(x)
-2	-6
-1	0
$-\frac{1}{3}$	4
0	2
1	-4

x	w(x)
-2	9
-1	0
$-\frac{1}{3}$	-6
0	-3
1	6



w is a vert. stretch of v by a factor of 3/2 & a reflection in the x-axis

HW 1.1b

(Pg. 8)

#34, 37, 40, 41, 43, 48, 51, 54, 56, 61

1.2 Vocab

1. PIECEWISE FUNCTION: a **PIECEWISE FUNCTION** is a Function defined by TWO OR MORE equations where each "piece" applies to a different part of its domain.

2. STEP FUNCTION: a **STEP FUNCTION** is a piecewise function defined by a constant value over each part of its domain. (looks like steps...or a staircase.)

1.2

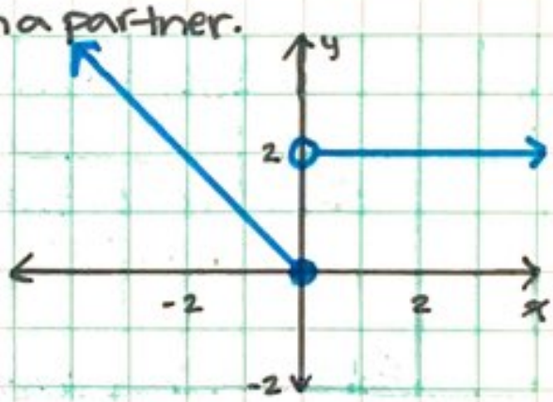
1.2 PIECEWISE Functions

Objective: Today we will evaluate, graph, & write piecewise functions & step functions.

Warm-up

EXPLORATION 1 Work with a partner.

(a) Does the graph represent y as a function of x? Justify your conclusion.



(b) What is the value of the function when $x=0$? How can you tell?

(c) Write an equation for the values when $x \leq 0$. $f(x) =$ _____ if $x \leq 0$.

(d) Write an equation for the values when $x > 0$. $f(x) =$ _____ if $x > 0$.

(e) Combine parts (c) & (d) to write a single description of the function.

$$f(x) = \begin{cases} \text{\_\_\_\_\_}, & \text{if } x \leq 0 \\ \text{\_\_\_\_\_}, & \text{if } x > 0 \end{cases}$$

Notes

PIECEWISE FUNCTIONS

↳ defined TWO OR MORE equations with NON-OVERLAPPING Domains.

↳ each equation is a "piece" of the function over a specific domain.

STEPS FOR EVALUATING PIECEWISE FUNCTIONS

1. Find which "piece" of the function that the x-value fits based on the domain.
2. Plug the x-value into that "piece" of the function ONLY & Evaluate!

Practice Problems

Evaluate the function.

$$f(x)=\begin{cases}5x-1, & \text{if } x<-2\\ x+3, & \text{if } x\geq -2\end{cases}$$

$$g(x)=\begin{cases}-x+4, & \text{if } x\leq -1\\ 3, & \text{if } -1<x<2\\ 2x-5, & \text{if } x\geq 2\end{cases}$$

(4) $f(-2)$: $-2=-2$ ⇓ $f(-2)=(-2)+3=1$

(6) $f(5)$: $5\geq -2$ ⇓ $f(5)=(5)+3=8$

(8) $g(-1)$: $-1=-1$ ⇓ $g(-1)=-(-1)+4=5$

(9) $g(0)$: $-1<0<2$ ⇓ $g(0)=3$

(11) $g(2)$: $2=2$ ⇓ $g(2)=2(2)-5=-1$

(14) The total cost (in dollars) of ordering x custom shirts is represented by the piecewise function

$$C(x)=\begin{cases}17x+20, & \text{if } 0\leq x<25\\ 15.80x+20, & \text{if } 25\leq x<50\\ 14x+20, & \text{if } x\geq 50.\end{cases}$$

Determine the total cost of ordering 26 shirts.

$25\leq 26<50 \Rightarrow C(26)=15.8(26)+20=\$430.80$

Graph the function. Describe the domain & range.

(17)

$$y=\begin{cases}3x-2, & \text{if } x\leq -1\\ x+2, & \text{if } x>-1\end{cases}$$

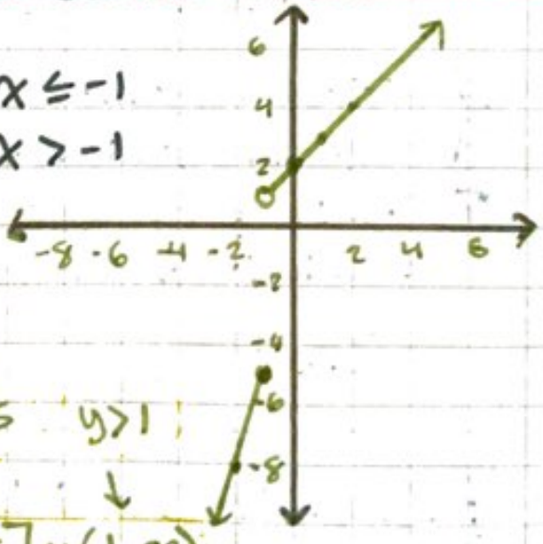
x	y
-2	-8
-1	-5

(● ; $3x-2$)

x	y
-1	1
0	2
1	3

(○ ; $x+2$)

$y\leq -5$ ↓ $y>1$ ↓



D: $\mathbb{R}$ R: $(-\infty,-5]\cup(1,\infty)$

Not the same as the book... I wrote it down wrong.

(19)

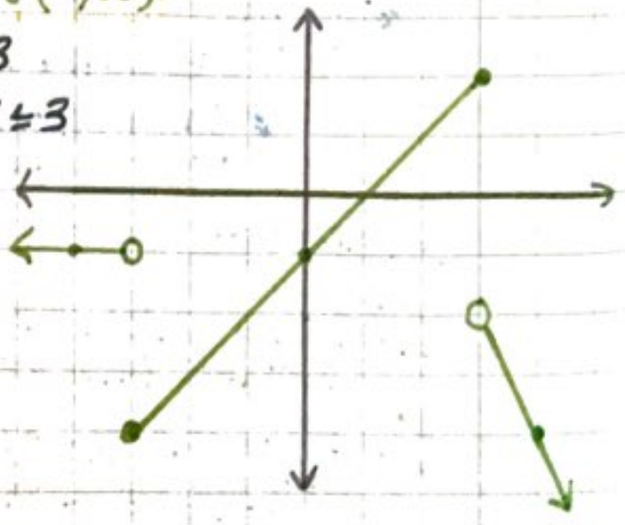
$$y=\begin{cases}-1, & \text{if } x<-3\\ x-1, & \text{if } -3\leq x\leq 3\\ -2x+4, & \text{if } x>3\end{cases}$$

x	y
-4	-1
-3	-1
-3	-4
0	-1
3	2
3	-2
4	-4

○ = Not included

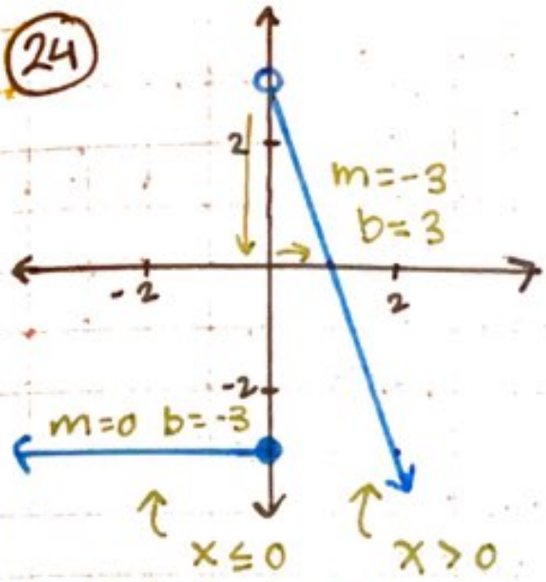
D: $\mathbb{R}$

R: $(-\infty,2]$ ↓ $y\leq 2$



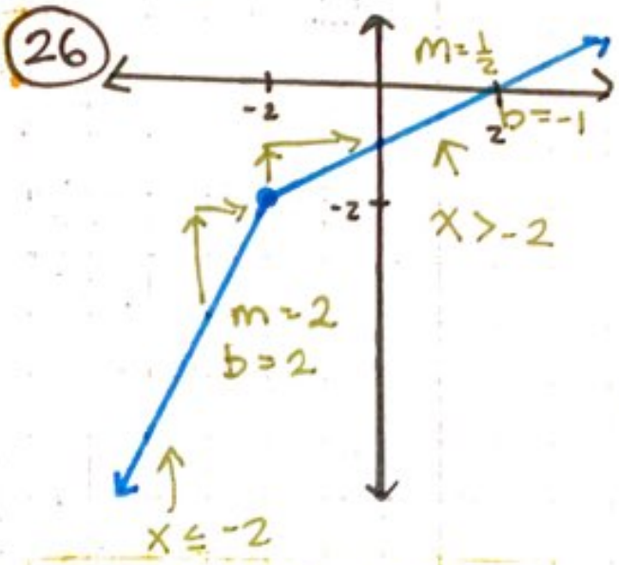
Write a piecewise function for the graph.

(24)



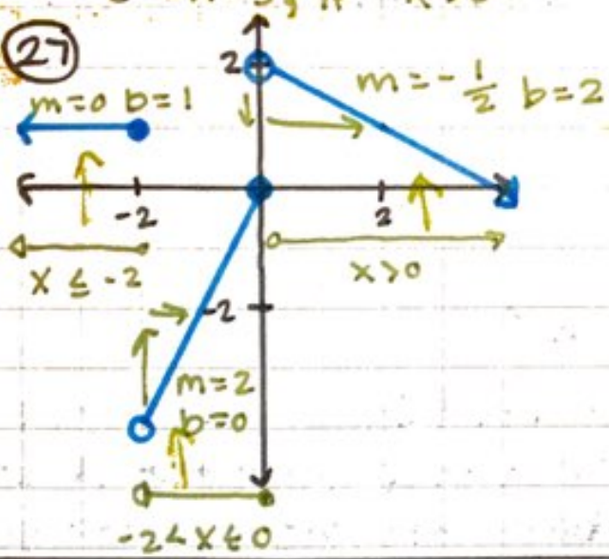
$$y = \begin{cases} -3, & \text{if } x \le 0 \\ -3x+3, & \text{if } x > 0 \end{cases}$$

(26)



$$y = \begin{cases} 2x+2, & \text{if } x \le -2 \\ \frac{1}{2}x-1, & \text{if } x > -2 \end{cases}$$

(27)



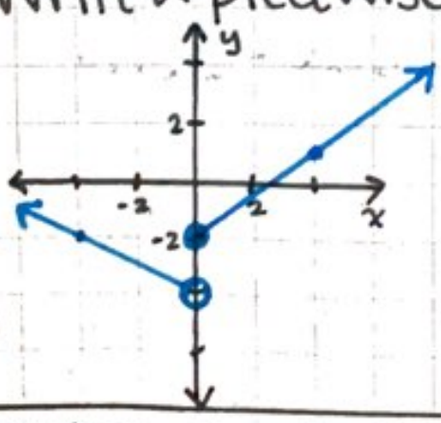
$$y = \begin{cases} 1, & \text{if } x \le -2 \\ 2x, & \text{if } -2 < x \le 0 \\ -\frac{1}{2}x+2, & \text{if } x > 0 \end{cases}$$

HW 1.2a
(Pg. 16)
#3,5,7,
10,12,13,
15,18,20,
23,25,28

1.2 Day 2

Warm-up

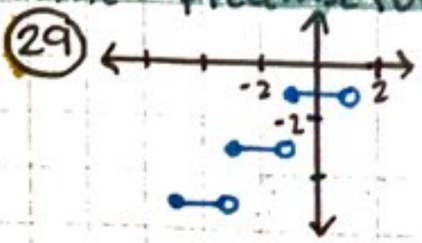
Write a piecewise function for the graph.



Notes/Practice Problems

Write a piecewise function for the graph.

(29)

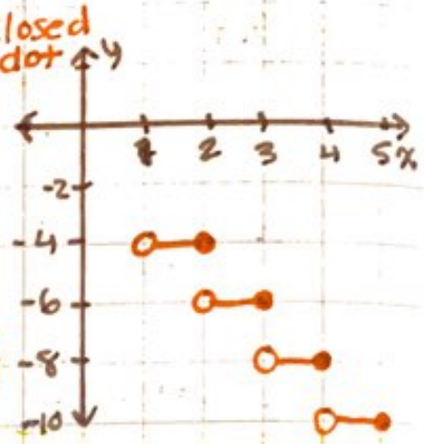


$$y = \begin{cases} -5, & \text{if } -5 \le x < -3 \\ -3, & \text{if } -3 \le x < -1 \\ -1, & \text{if } -1 \le x < 1 \end{cases}$$

Graph the STEP FUNCTION. Describe the domain & range.

○ = open dot ● = closed dot

(32)
$$f(x)=\begin{cases}-4, & \text{if } 1<x\le 2\\ -6, & \text{if } 2<x\le 3\\ -8, & \text{if } 3<x\le 4\\ -10, & \text{if } 4<x\le 5\end{cases}$$

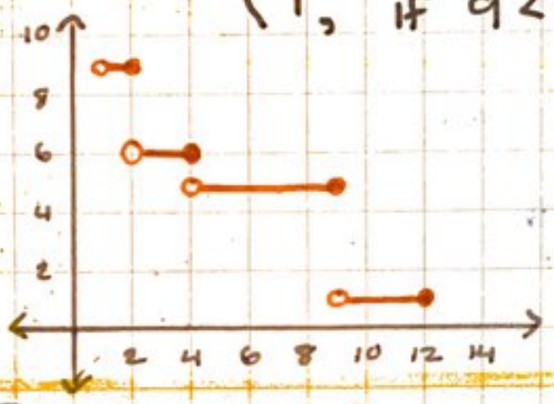


D: $1<x\le 5$ R: $\{y \mid -4,-6,-8,-10\}$

(33)
$$f(x)=\begin{cases}9, & \text{if } 1<x\le 2\\ 6, & \text{if } 2<x\le 4\\ 5, & \text{if } 4<x\le 9\\ 1, & \text{if } 9<x\le 12\end{cases}$$

D: $1<x\le 12$

R: $\{1, 5, 6, 9\}$

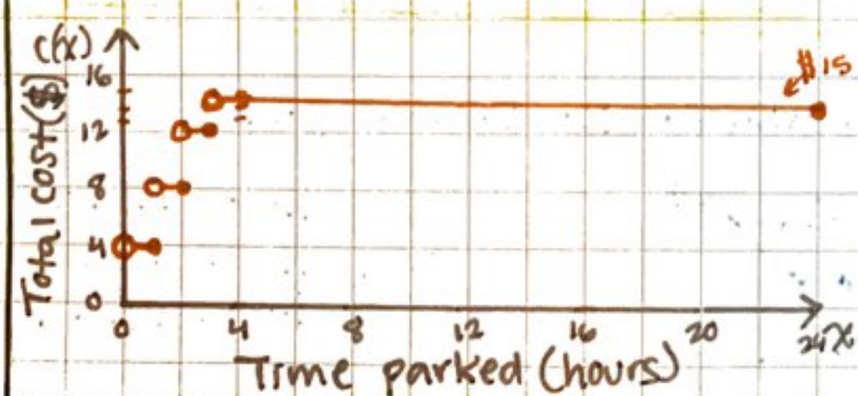


(36) The rates for a parking garage are shown. Write and graph a step function that represents the relationship between the number x of hours a car is parked in the garage & the total cost of parking in the garage for 1 day.

Daily Parking Garage Rates
$4 per hour and
$15 daily maximum

$$C(x)=\begin{cases}4, & \text{if } 0<x\le 1\\ 8, & \text{if } 1<x\le 2\\ 12, & \text{if } 2<x\le 3\\ 15, & \text{if } 3<x\le 24\end{cases}$$

Note: 1 day = 24 hrs.
So Domain: $0<x\le 24$



Write the absolute value function as a piecewise function.

Left side: $x < h$ & change |abs| into parentheses & distribute -1 into it.

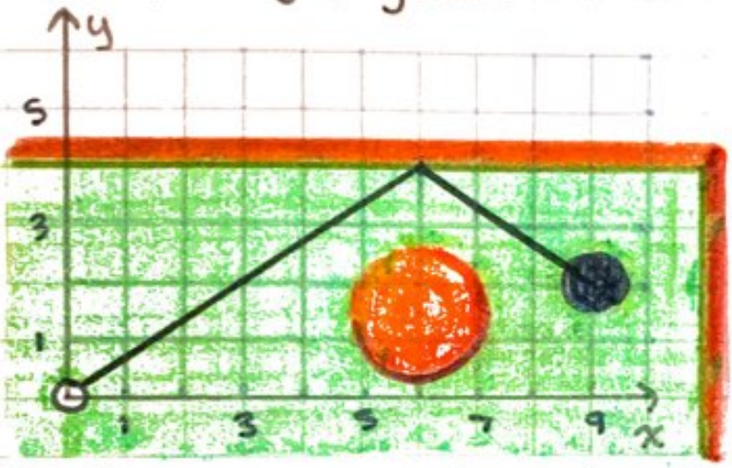
Right side: $x \geq h$ & treat |abs| as parentheses

(37) $y = |x| + 1$
V: (0, 1)
$y = \begin{cases} -x+1, & x < 0 \\ x+1, & x \geq 0 \end{cases}$

(42) $y = 4|x-1|$
V: (1, 0)
$y = \begin{cases} -4x+4, & x < 1 \\ 4x-4, & x \geq 1 \end{cases}$

(45) $y = -|x-3|+2$
V: (3, 2)
$y = \begin{cases} x-1, & x < 3 \\ -x+5, & x \geq 3 \end{cases}$

(48) You are trying to make a hole in one on the miniature golf green.



$-a \Rightarrow$ opens down
V: (6, 4)
Left: $x < 6$, $m = \frac{2}{3}$, $b = 0$
Right: $x \geq 6$, $m = -\frac{2}{3}$, $b = 8$

(a) Write an absolute value function that represents the path of the ball.

$y = -\frac{2}{3}|x-6| + 4$

(b) Write the function in part (a) as a piecewise function.

$y = \begin{cases} \frac{2}{3}x, & \text{if } x < 6 \\ -\frac{2}{3}x + 8, & \text{if } x \geq 6 \end{cases}$

HW 1.2b
(Pg. 16)
#30, 31, 34, 35, 38, 41, 46, 47, 49, 50

Review your Notes, classwork, & HW & then answer the following questions:

- What do I NEED to RELEARN/STUDY FIRST?
- What do I THINK I KNOW, but have questions about still?
- What do I KNOW & could teach to someone else?

Stoplight Reflection

1.3. Vocab.

1. INVERSE RELATION: an INVERSE RELATION switches the input & output values of the original relation.