4.2 Solving Quadratics by Graphing

Objective: Today we will solve quadratic equations by graphing and we will use graphs to find & approximate zeros.

Warm-up: Simplify the expression.

1. $\sqrt{144x^3}$
2. $\sqrt[3]{-54x^3}$
3. $\dfrac{7}{\sqrt{2}+3}$
4. $4\sqrt{5} - \sqrt{3} + 3\sqrt{5}$
5. $\sqrt{3}\,(\sqrt{2} - \sqrt{8})$

4.2 Solving Quadratic Equations by Graphing

Method 1

Solving Quadratic Equations by Graphing

1. Write the equation in Standard Form: $ax^2+bx+c=0$.
2. Graph $y=ax^2+bx+c$.
3. Find the x-intercepts, if any.

Method 2

Graph Each Side of the Equation

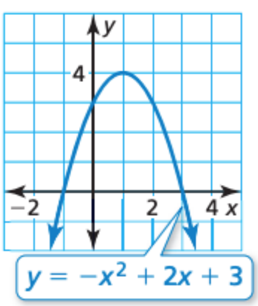
When the equation is NOT in Standard Form,
Let
1. $y=$ "Left side" & $y=$ "Right side".
2. Graph both equations together.
3. Find the Points of Intersection & the solutions are the x-coordinates.

Number of Solutions (Graphically)

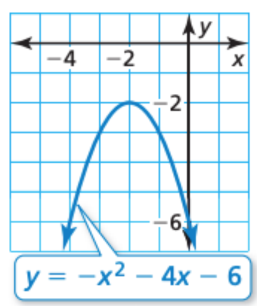
- 2 Real = 2 x-intercepts
- 1 Real = 1 x-intercept (also the Vertex)
- No Real Sol. = No x-intercepts

In Exercises 5–8, use the graph to solve the equation.

5. $-x^2 + 2x + 3 = 0$



8. $-x^2 - 4x - 6 = 0$



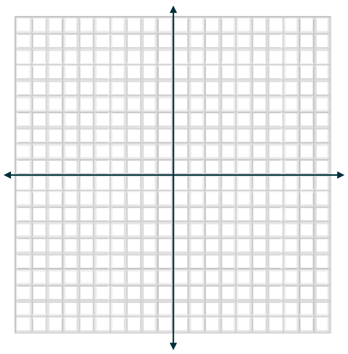
In Exercises 9–12, write the equation in standard form.

10. $-x^2 = 15$

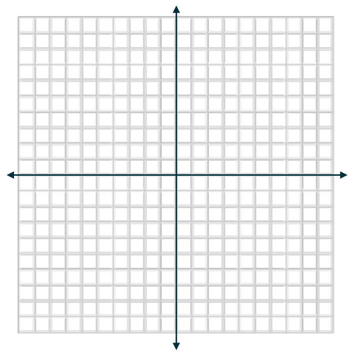
11. $2x - x^2 = 1$

In Exercises 13–24, solve the equation by graphing.
(See Examples 1, 2, and 3.)

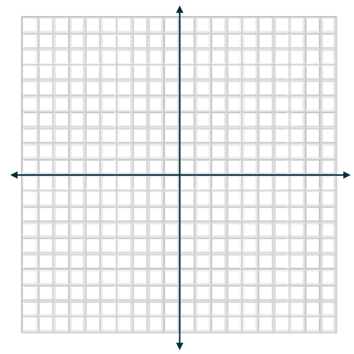
16. $x^2 - 6x - 7 = 0$



17. $x^2 = 6x - 9$



22. $5x - 6 = x^2$



28. **MODELING WITH MATHEMATICS** The height h (in feet) of an underhand volleyball serve can be modeled by $h = -16t^2 + 30t + 4$, where t is the time (in seconds).

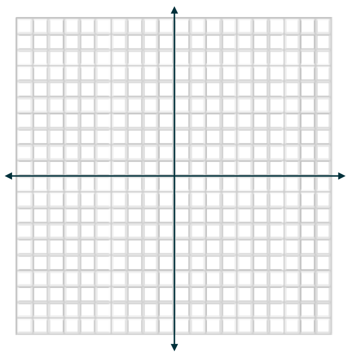
a. Do both t-intercepts of the graph of the function have meaning in this situation? Explain.

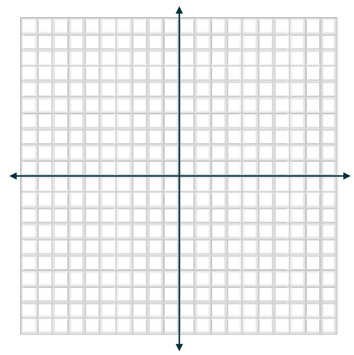
b. No one receives the serve. After how many seconds does the volleyball hit the ground?

In Exercises 29–36, solve the equation by using Method 2 from Example 3.

32. $x^2 = 6x - 5$

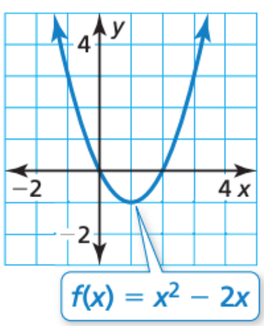
35. $-x^2 - 5 = -2x$





In Exercises 37–42, find the zero(s) of *f*.

37.



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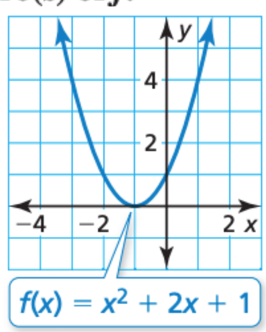

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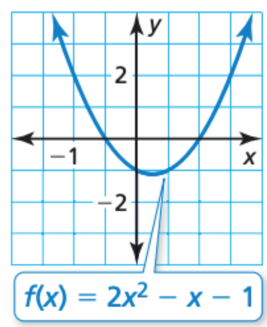
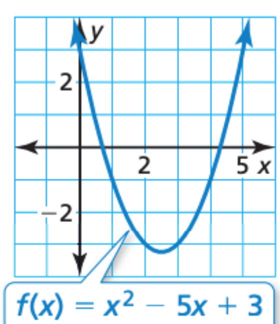


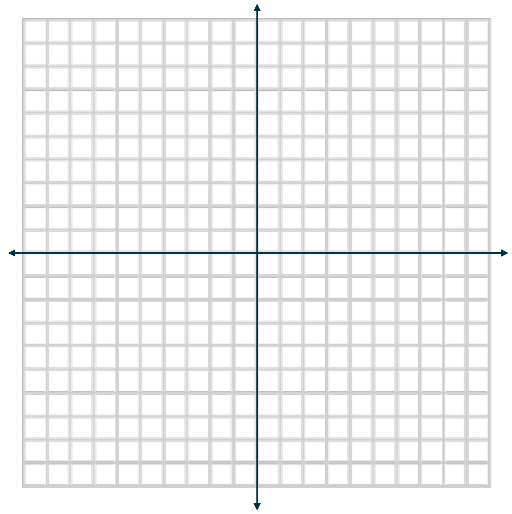
In Exercises 43–46, approximate the zeros of *f* to the nearest tenth. *(See Example 5.)*

43.



In Exercises 47–52, graph the function. Approximate the zeros of the function to the nearest tenth, if necessary.

49. $y = -x^2 + 4x - 2$



53. **MODELING WITH MATHEMATICS** At a Civil War reenactment, a cannonball is fired into the air with an initial vertical velocity of 128 feet per second. The release point is 6 feet above the ground. The function $h = -16t^2 + 128t + 6$ represents the height h (in feet) of the cannonball after t seconds. *(See Example 6.)*

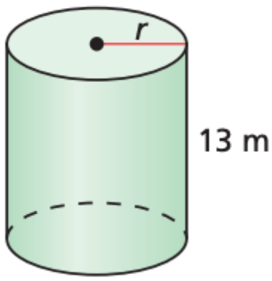
a. Find the height of the cannonball each second after it is fired.

b. Use the results of part (a) to estimate when the height of the cannonball is 150 feet.

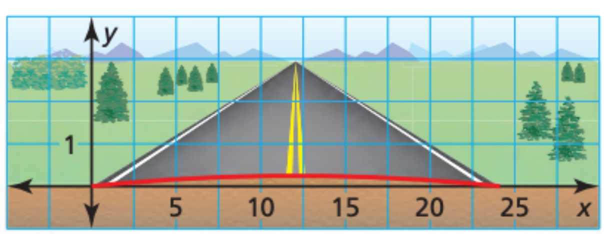
c. Using a graph, after how many seconds is the cannonball 150 feet above the ground?

MATHEMATICAL CONNECTIONS **In Exercises 55 and 56, use the given surface area S of the cylinder to find the radius r to the nearest tenth.**

56. $S = 750 \text{ m}^2$



61. **MODELING WITH MATHEMATICS** To keep water off a road, the surface of the road is shaped like a parabola. A cross section of the road is shown in the diagram. The surface of the road can be modeled by $y = -0.0017x^2 + 0.041x$, where x and y are measured in feet. Find the width of the road to the nearest tenth of a foot.



4.3 Solving Quadratics Using Square Roots

Objective: Today we will solve quadratic equations using square roots.

Checkpoint: Exploration 1

Work with a partner. Solve each equation by graphing. Explain how the number of solutions of $ax^2 + c = 0$ relates to the graph of $y = ax^2 + c$.

a. $x^2 - 4 = 0$ **b.** $2x^2 + 5 = 0$

c. $x^2 = 0$ **d.** $x^2 - 5 = 0$

4.3 SOLVING QUADRATIC EQUATIONS using SQUARE ROOTS

SOLUTIONS of $x^2 = d$

- 2 REAL Solutions when $d > 0$
  - ↳ $x = \pm\sqrt{d}$
- 1 REAL Solution when $d = 0$
  - ↳ $x = 0$
- NO REAL Solutions when $d < 0$

In Exercises 3–8, determine the number of real solutions of the equation. Then solve the equation using square roots.

5. $x^2 = -21$ **6.** $x^2 = 400$ **7.** $x^2 = 0$

In Exercises 9–18, solve the equation using square roots.

11. $3x^2 + 12 = 0$ **15.** $-3x^2 - 5 = -5$

12. $x^2 - 55 = 26$ **18.** $9x^2 - 35 = 14$

In Exercises 19–24, solve the equation using square roots. *(See Example 2.)*

21. $(2x - 1)^2 = 81$

23. $9(x + 1)^2 = 16$

In Exercises 25–30, solve the equation using square roots. Round your solutions to the nearest hundredth. *(See Example 3.)*

27. $2x^2 - 9 = 11$

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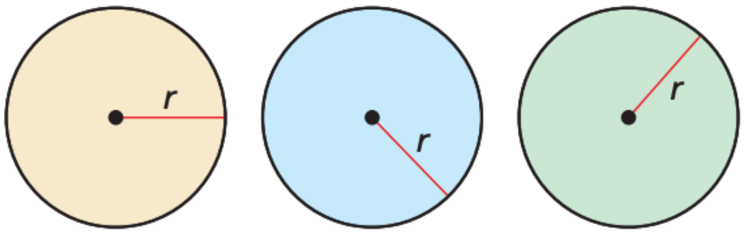
32. **MODELING WITH MATHEMATICS** An in-ground pond has the shape of a rectangular prism. The pond has a depth of 24 inches and a volume of 72,000 cubic inches. The length of the pond is two times its width. Find the length and width of the pond. *(See Example 4.)*

36. **MATHEMATICAL CONNECTIONS** The area A of a circle with radius r is given by the formula $A = \pi r^2$. *(See Example 5.)*

a. Solve the formula for r.

b. Use the formula from part (a) to find the radius of each circle.



$A = 113\ \text{ft}^2$ $A = 1810\ \text{in.}^2$ $A = 531\ \text{m}^2$

c. Explain why it is beneficial to solve the formula for r before finding the radius.

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42. **THOUGHT PROVOKING** The quadratic equation

$$ax^2 + bx + c = 0$$

can be rewritten in the following form.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Use this form to write the solutions of the equation.